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A fractional differential inequality with application

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ABSTRACT

We establish a fractional differential inequality using desingularization techniques combined with some generalizations of algebraic Bihari-type inequalities. We use this inequality to prove global existence and determine the asymptotic behavior of solutions for a family of fractional differential equations.

1. Introduction

In this work we establish a boundedness result for solutions of the fractional differential inequality

$$[D_a^{\alpha} u(t)]^2 \le a(t) + \sum_{i=1}^{k} b_i(t) D_a^{\beta_i} u(t), \tag{1}$$

with $0 \le \beta_1 < \beta_2 < \cdots < \beta_k < \alpha < 1$.

Inequalities play an important role in the qualitative analysis of solutions of differential and integral equations. They are used in the study of existence, uniqueness, boundedness, stability, continuous dependence, and perturbation.

In [1] we established Bihari-type inequalities involving fractional derivatives. We showed how to use these inequalities to prove boundedness and global existence, and to determine the asymptotic behavior for some families of fractional differential equations. In [2] we established some nonlinear differential inequalities involving fractional derivatives and provided applications illustrating the usefulness of these inequalities.

In this paper we obtain a bound for the highest derivative satisfying the inequality (1). We also show how to use this bound in analyzing a family of fractional differential equations.

The results obtained may be seen as generalizations and extensions of analogous results from the integer order case (which may be found in [3,4]) to the fractional order case. Our results may be applied, for instance, to differential equations of the form

$$D_a^{\alpha} u(t) = f(t, u, \{D_a^{\beta_i} u(t)\}_{i=1}^{k}),$$

where α and β_i are not necessarily integers and f could be a nonlinear function, namely, when f takes the form of or bounded by the right-hand side in (1). For well posedness of such (Cauchy) problems we refer the reader to [5–9]. The special case

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