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Multiple solutions for a Neumann system involving subquadratic nonlinearities

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ABSTRACT

In this paper, we consider the model semilinear Neumann system

$$\begin{cases} -\Delta u + a(x)u = \lambda c(x)F_u(u,v) & \text{in } \Omega, \\ -\Delta v + b(x)v = \lambda c(x)F_v(u,v) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \qquad (N_\lambda)$$

where $\Omega \subset \mathbb{R}^N$ is a smooth open bounded domain, ν denotes the outward unit normal to $\partial\Omega$, $\lambda \geq 0$ is a parameter, $a, b, c \in L^\infty_+(\Omega) \setminus \{0\}$, and $F \in C^1(\mathbb{R}^2, \mathbb{R}) \setminus \{0\}$ is a nonnegative function which is subquadratic at infinity. Two nearby numbers are determined in explicit forms, $\underline{\lambda}$ and $\overline{\lambda}$ with $0 < \underline{\lambda} \leq \overline{\lambda}$, such that for every $0 \leq \lambda < \underline{\lambda}$, system (N_λ) has only the trivial pair of solution, while for every $\lambda > \overline{\lambda}$, system (N_λ) has at least two distinct nonzero pairs of solutions.

1. Introduction

Let us consider the quasilinear Neumann system

$$\begin{cases} -\Delta_p u + a(x)|u|^{p-2}u = \lambda c(x)F_u(u,v) & \text{in } \Omega, \\ -\Delta_q v + b(x)|v|^{q-2}v = \lambda c(x)F_v(u,v) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \qquad (N_\lambda^{p,q})$$

where $p, q > 1$; $\Omega \subset \mathbb{R}^N$ is a smooth open bounded domain; ν denotes the outward unit normal to $\partial\Omega$; $a, b, c \in L^\infty(\Omega)$ are some functions; $\lambda \geq 0$ is a parameter; and F_u and F_v denote the partial derivatives of $F \in C^1(\mathbb{R}^2, \mathbb{R})$ with respect to the first and second variables, respectively.

Recently, problem $(N_\lambda^{p,q})$ has been considered by several authors. For instance, under suitable assumptions on a, b, c and F, El Manouni and Kbiri Alaoui [1] proved the existence of an interval $A \subset (0, \infty)$ such that $(N_\lambda^{p,q})$ has at least three solutions whenever $\lambda \in A$ and $p, q > N$. Lisei and Varga [2] also established the existence of at least three solutions for the system $(N_\lambda^{p,q})$ with nonhomogeneous and nonsmooth Neumann boundary conditions. Di Falco [3] proved the existence of infinitely many solutions for $(N_\lambda^{p,q})$ when the nonlinear function F has a suitable oscillatory behavior. Systems similar to $(N_\lambda^{p,q})$ with the Dirichlet boundary conditions were also considered by Afrouzi and Heidarkhani [4,5], Boccardo and de Figueiredo [6], Heidarkhani and Tian [7], and Li and Tang [8]; see also references therein.

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