Nonlinear Analysis 74 (2011) 1814–1822

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Weak and strong convergence theorems for nonspreading-type mappings in Hilbert spaces

M.O. Osilike *, F.O. Isiogugu

Department of Mathematics, University of Nigeria, Nsukka, Nigeria

ARTICLE INFO

Article history:
Received 20 July 2010
Accepted 26 October 2010

MSC:
47A35
47J05
47J25
47H09

Keywords:
Hilbert spaces
k-strictly pseudononspreading mappings
Nonspreading mappings
Fixed points
Mean ergodic theorem
Weak convergence
Strong convergence

ABSTRACT

Weak and strong convergence theorems are proved in real Hilbert spaces for a new class of nonspreading-type mappings more general than the class studied recently in Kurokawa and Takahashi [Y. Kurokawa, W. Takahashi, Weak and strong convergence theorems for nonspreading mappings in Hilbert spaces, Nonlinear Anal. 73 (2010) 1562–1568]. We explored an auxiliary mapping in our theorems and proofs and this also yielded a strong convergence theorem of Halpern's type for our class of mappings and hence resolved in the affirmative an open problem posed by Kurokawa and Takahashi in their final remark for the case where the mapping T is averaged.

1. Introduction

Let H be a real Hilbert space. A mapping $T : D(T) \subseteq H \to H$ is said to be *L-Lipschitzian* if there exists $L > 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\|, \quad \forall x, y \in D(T). \tag{1.1}$$

If $L < 1$ in (1.1), T is said to be *strictly contractive*, while T is said to be *nonexpansive* if $L = 1$. T is said to be *quasi-nonexpansive* if $F(T) = \{x \in D(T) : Tx = x\} \neq \emptyset$ and $\|Tx - p\| \leq \|x - p\|$ for all x in $D(T)$ and for all p in $F(T)$. Furthermore, T is said to be *firmly nonexpansive* if

$$\|Tx - Ty\|^2 \leq \langle x - y, Tx - Ty\rangle, \quad \forall x, y \in D(T).$$

Every nonexpansive mapping with a nonempty fixed point set $F(T)$ is quasi-nonexpansive, and firmly nonexpansive mappings are important examples of nonexpansive mappings.

Recently, Kohsaka and Takahashi [1,2] introduced an important class of mappings which they called the class of *nonspreading mappings*. Let E be a real smooth, strictly convex and reflexive Banach space, and let j denote the duality mapping of E. Let C be a nonempty closed convex subset of E. They called a mapping $T : C \to C$ nonspreading if

$$\phi(Tx, Ty) + \phi(Ty, Tx) \leq \phi(Tx, y) + \phi(Ty, x) \tag{1.2}$$

* Corresponding author.
E-mail addresses: osilike@yahoo.com, micah.osilike@unn.edu.ng (M.O. Osilike), obifeli2001@yahoo.com (F.O. Isiogugu).

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.10.054