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# Stability of semi-infinite vector optimization problems without compact constraints

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ABSTRACT

This paper is devoted to the continuity of solution maps for perturbation semi-infinite vector optimization problems without compact constraint sets. The sufficient conditions for lower semicontinuity and upper semicontinuity of solution maps under functional perturbations of both objective functions and constraint sets are established. Some examples are given to analyze the assumptions in the main result.



## 1. Introduction

Let $T$ be a nonempty compact subset of a Hausdorff topological space. $C[T, \mathbb{R}^l]$ denotes the set of all continuous vector functions $b : T \to \mathbb{R}^l$ with the norm defined as follows:

$$\|b\| := \max_{t \in T} \|b(t)\|_l, \quad b \in C[T, \mathbb{R}^l],$$

where $\|\cdot\|_l$ denotes the Euclidean norm in the $l$-dimensional real space $\mathbb{R}^l$.

Given a closed convex pointed cone $K_m \subset \mathbb{R}^m$ with a nonempty interior, the partial order $\preceq_{K_m}$ ($\prec_{K_m}$) in $\mathbb{R}^m$ is defined as $y \preceq_{K_m} y'$ ($y \preceq_{K_m} y'$) if and only if $y' - y \in K_m$ ($y' - y \in int(K_m)$, respectively) for $y, y' \in \mathbb{R}^m$.

$f : \mathbb{R}^n \to \mathbb{R}^m$ is said to be a $K_m$-*convex* vector function if

$$f(\lambda x_1 + (1-\lambda)x_2) \preceq_{K_m} \lambda f(x_1) + (1-\lambda) f(x_2)$$

for all $x_1, x_2 \in \mathbb{R}^n$ and all $\lambda \in [0, 1]$.

Let $B_i := \{x \in \mathbb{R}^n : \|x\|_n \leq i\}, i = 1, 2, \ldots$. Then $\{B_i\}_{i=1}^{\infty}$ is the sequence of compact sets in $\mathbb{R}^n$ satisfying $B_i \subset int\, B_{i+1}$ and $\mathbb{R}^n = \bigcup_{i=1}^{\infty} B_i$. $CO_{K_m}[\mathbb{R}^n, \mathbb{R}^m]$ denotes the set of all continuous $K_m$-convex vector functions $f : \mathbb{R}^n \to \mathbb{R}^m$ with the metric defined as follows:

$$\varrho(f^1, f^2) = \sum_{i=1}^{\infty} \frac{1}{2^i} \frac{\varrho_i(f^1, f^2)}{1 + \varrho_i(f^1, f^2)}, \quad f^1, f^2 \in CO_{K_m}[\mathbb{R}^n, \mathbb{R}^m]$$

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