Nonlinear Analysis 74 (2011) 1402–1409

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Blow-up of solutions for a nonlinear beam equation with fractional feedback

Soraya Labidi [a], Nasser-eddine Tatar [b,*]

[a] *Laboratoire de Mathématiques Appliquées, Université Badji Mokhtar, B. P. 12, Annaba 23000, Algeria*

[b] *King Fahd University of Petroleum and Minerals, Department of Mathematics and Statistics, Dhahran 31261, Saudi Arabia*

ARTICLE INFO

Article history:
Received 8 May 2010
Accepted 4 October 2010

MSC:
35L10
93B30

Keywords:
Blow up
Boundary feedback
Fractional derivative
Non-dissipative system
Singular kernel

ABSTRACT

A nonlinear beam equation describing the transversal vibrations of a beam with boundary feedback is considered. The boundary feedback involves a fractional derivative. We discuss the asymptotic behavior of solutions. In fact, we prove that solutions blow up in finite time under certain assumptions on the nonlinearity.

1. Introduction and description of the model

In this paper we study the behavior of solutions to the following nonlinear beam equation with fractional damping at the boundary

$$\begin{cases} u_{tt} + \Delta^2 u + \Delta g(\Delta u) = 0, \quad x \in \Omega, t > 0 \\ u = \dfrac{\partial u}{\partial \nu} = 0, \quad x \in \Gamma_1,\ t > 0 \\ \Delta u = 0, \quad x \in \Gamma_0,\ t > 0 \\ \dfrac{\partial \Delta u}{\partial \nu} = \dfrac{c}{\Gamma(\beta)} \displaystyle\int_0^t (t-s)^{\beta-1} u_t ds - au, \quad x \in \Gamma_0,\ t > 0 \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad x \in \Omega \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbf{R}^n$ with smooth boundary $\partial\Omega$. The boundary is divided into Γ_0 and Γ_1 in such a way that $\partial\Omega = \Gamma_0 \cup \Gamma_1$, $\Gamma_0 \cap \Gamma_1 = \emptyset$ and $\lambda_{n-1}(\Gamma_1) > 0$ where λ_{n-1} denotes the $(n-1)$-dimensional Lebesgue measure on the boundary $\partial\Omega$. The initial data $u_0(x)$ and $u_1(x)$ are given functions, $g(s)$ is a given nonlinear function, $\partial/\partial\nu$ denotes the outward normal derivative and $\Gamma(.)$ is the usual Euler gamma function. The constants a and c are positive and the power β in the integral term is such that $0 < \beta < 1$.

This problem is known as the Euler–Bernoulli beam problem and describes the transversal vibrations of a beam. Here the control is a torque applied on a part of the boundary of the beam. The integral term in the boundary condition is a time fractional derivative of u of order $1-\beta$. It represents a boundary feedback which helps reduce the effect of the reflected waves. In fact, it is a boundary damping. Therefore, it is important to have an idea of the sufficient conditions which make

* Corresponding author. Tel.: +966 3 860 4654; fax: +966 3 860 2340.
E-mail addresses: labidis1@yahoo.fr (S. Labidi), tatarn@kfupm.edu.sa (N.-e. Tatar).

0362-546X/$ – see front matter
doi:10.1016/j.na.2010.10.012