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Energy bound for sign changing solutions of an asymptotically linear elliptic equation in $\mathbb{R}^N$ ☆

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ABSTRACT

In this paper, we give an estimate of a lower bound for the energy of sign changing solutions of the following nonlinear elliptic equation

$$-\Delta u + u = f(u), \quad u \in H^1(\mathbb{R}^N), \tag{$*$}$$

where f is locally Lipschitz continuous and asymptotically linear.

Weth (2006) [5] gave a lower bound for the energy of sign changing solutions of Eq. ($*$) if f is superlinear. Here, we show that the result of Weth (2006) [5] is also true if f is asymptotically linear.

The new ingredient of this problem is to prove that the energy of a mountain pass solution is equal to the ground state energy (see Lemma 2.1), which is obvious when f is superlinear but nontrivial when f is asymptotically linear.

1. Introduction

In this paper, we consider the following nonlinear elliptic equation

$$\begin{cases} -\Delta u + u = f(u), & x \in \mathbb{R}^N \\ u \in H^1(\mathbb{R}^N). \end{cases} \tag{1.1}$$

This equation is applied in many fields, such as in nonlinear optics and plasma physics (see [1,2] for example). In particular, solutions of (1.1) correspond to standing waves of nonlinear time dependent Schrödinger equations.

Throughout this paper, we always assume that the nonlinearity $f(u)$ satisfies the following conditions:

(f1) $f : \mathbb{R} \to \mathbb{R}$ is locally Lipschitz continuous, $f(0) = 0$ and

$$\lim_{u,v\to 0, u\neq v} \frac{f(u) - f(v)}{u - v} = 0. \tag{1.2}$$

(f2) There exists $l \in (1, +\infty)$ such that

$$\lim_{|u|\to+\infty} \frac{f(u)}{u} = l. \tag{1.3}$$

(f3) The function $u \mapsto \frac{f(u)}{|u|}$ is strictly increasing on $\mathbb{R} \setminus \{0\}$.

(f4) f is odd in u.

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