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On the regularity criterion of axisymmetric weak solutions to the 3D Navier–Stokes equations

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ABSTRACT

In this paper, we consider the regularity criterion of axisymmetric weak solutions to the Navier–Stokes equations in $\mathbb{R}^3$. Let u be an axisymmetric weak solution in $\mathbb{R}^3 \times (0, T)$, $w = \operatorname{curl} u$, and w^θ be the azimuthal component of w in the cylindrical coordinates. It is proved that u becomes a regular solution if $w^\theta \in L^{\frac{2}{2-s}}(0, T; \dot{\mathcal{M}}_{2,\frac{3}{s}})$, where $\dot{\mathcal{M}}_{2,\frac{3}{s}}$ is the critical Morrey–Campanato space.

1. Introduction

We consider the following Cauchy problem for the incompressible Navier–Stokes equations in $\mathbb{R}^3 \times (0, T)$:

$$\begin{aligned}
&\partial_t u + u \cdot \nabla u - \Delta u = -\nabla p, \quad (x, t) \in \mathbb{R}^3 \times (0, T), \\
&\operatorname{div} u = 0, \quad (x, t) \in \mathbb{R}^3 \times (0, T), \\
&u(x, 0) = u_0(x), \quad x \in \mathbb{R}^3,
\end{aligned} \tag{1.1}$$

where $u = (u^1(x, t), u^2(x, t), u^3(x, t))$ is the velocity field and $p = p(x, t)$ is the scalar pressure, while $= (u_0^1(x), u_0^2(x), u_0^3(x))$ is a given initial velocity vector satisfying $\nabla \cdot u_0 = 0$. Here we use the notation:

$$u \cdot \nabla v = \sum_{i=1}^{3} u^i \frac{\partial v}{\partial x_i}, \qquad \nabla \cdot u = \sum_{i=1}^{3} \frac{\partial u^i}{\partial x_i},$$

for vector functions u, v.

Although a global weak solution of (1.1) was first constructed by Leray [1] in 1934, the fundamental problem on uniqueness and regularity of weak solutions still remains open, although great contributions have been made in an effort to understand the regularities of the weak solution. It is well known that regularity can be persistent under certain conditions, which was introduced in the celebrated work of Serrin [2], and can be described as follows (see also [3]).

A weak solution u is regular if the growth condition

$$u \in L^q(0, T; L^r) \quad \text{with } \frac{2}{q} + \frac{3}{r} \leq 1, 3 < r \leq \infty, \tag{1.2}$$

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