



Existence results of semilinear differential equations with nonlocal initial conditions in Banach spaces[☆]

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ABSTRACT

In this paper, we establish the existence results for semilinear differential systems with nonlocal initial conditions in Banach spaces. The approaches used are fixed point theorems combined with convex-power condensing operators. The first result obtained will be applied to a class of semilinear parabolic equations.

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1. Introduction

This paper is devoted to the following semilinear nonlocal problem

$$\begin{cases} u'(t) = Au(t) + f(t, u(t)), & t \in (0, T], \\ u(0) = g(u), \end{cases} \quad (1.1)$$

where $A : D(A) \subseteq X \rightarrow X$ is the infinitesimal generator of strongly continuous semigroup $S(t)$ for $t > 0$ in a real Banach space X , f and g are X -valued functions to be given later.

Many authors have studied the nonlocal Cauchy problem because it has better effect in the applications than the classical initial condition. We begin by mentioning some of the previous work done in the literature. This study was first considered by Byszewski. Byszewski and Lakshmikantham [1,2] proved the existence and uniqueness of mild solutions for nonlocal semilinear differential equations when f and g satisfy Lipschitz type conditions. In [3], Ntouyas and Tsamatos studied the case with compactness conditions. Subsequently, Byszewski and Akca [4] established the existence of solution to functional-differential equations when the semigroup is compact, and g is convex and compact on a given ball. In [5], Fu and Ezzinbi studied neutral functional-differential equations with nonlocal conditions. Benchohra and Ntouyas [6] discussed second order differential equations under compact conditions. For more details on the nonlocal problem, we refer to the papers of [3,7–19] and references therein.

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