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Homotopy method for solving ball-constrained variational inequalities

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ABSTRACT

In this paper, we present a smoothing homotopy method for solving ball-constrained variational inequalities by utilizing a similar Chen–Harker–Kanzow–Smale function to smooth Robinson's normal equation. Without any monotonicity condition on the defining map F, for the starting point chosen almost everywhere in R^n, the existence and convergence of the homotopy pathway are proven. Numerical experiments illustrate that the method is feasible and effective.

1. Introduction

Consider the variational inequality problem (VIP for abbreviation): find $y^* \in X$ such that

$$(y-y^*)^T F(y^*) \geq 0, \quad \text{for all } y \in X \tag{1}$$

where X is a nonempty closed subset of R^n and $F : \mathcal{D} \to R^n$ is continuously differentiable on some open set $\mathcal{D}$. In this paper, unless otherwise stated, we assume that

$$X := \left\{ y \in R^n \mid \|y\| \leq r \right\}, \tag{2}$$

where $r > 0$ and $\|\cdot\|$ is the Euclidean norm. Then (1) becomes a ball-constrained variational inequality problem (BVIP for abbreviation). It is well known if X is a closed convex subset of R^n, then solving (1) is equivalent to solving the following Robinson's normal equation

$$E(x) := F(\Pi_X(x)) + x - \Pi_X(x) = 0 \tag{3}$$

where for any $x \in R^n$, $\Pi_X(x)$ is the Euclidean projection of x onto X. In this sense if $x^* \in R^n$ is a solution of (3), then $y^* = \Pi_X(x^*)$ is a solution of (1), and conversely, if y^* is a solution of (1), then $x^* = y^* - F(y^*)$ is a solution of (3) [1]. (3) is a nonsmooth equation and has led to various generalized Newton methods. For a review of these methods, one can refer to [2–4]. Particularly, if we assume that X is of the form (2), then the Euclidean projection of x onto X becomes

$$\Pi_X(x) = \begin{cases} \dfrac{rx}{\|x\|} & \text{if } \|x\| > r, \\ x & \text{if } \|x\| \leq r. \end{cases}$$

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