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Average locally uniform rotundity and a class of nonlinear maps

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ABSTRACT

We consider some topological characterizations of dual Banach spaces that admit an equivalent dual average locally uniformly rotund norm and provide a criterion for such renorming which involves the class of σ-slicely continuous maps.

1. Introduction and preliminaries

In this note, we present some covering type characterizations of the class of Banach spaces that admit an equivalent renorming with dual average locally uniformly rotund norm, a property closely related with the notion of locally uniform rotundity. Using these results, we establish a criterion for the existence of such renormings in terms of the class of σ-slicely continuous maps, recently introduced in [1], where a nonlinear transfer method for locally uniformly rotund renorming has been developed. As an application of this criterion, we obtain a 3-space like result for dual average locally uniform rotundity.

Let us recall some definitions and terminology. All the Banach spaces considered in this note are real. The space X (or a norm $\|\cdot\|$ on X) is said to be *locally uniformly rotund* (LUR for short) if for every $x \in X$ and every sequence $(x_n)_n \subset X$ such that $\lim_n \|x_n\| = \|x\|$ and $\lim_n \|(x_n + x)/2\| = \|x\|$ we have $\lim_n \|x_n - x\| = 0$.

Obviously, if X is a LUR space, then X is *strictly convex*, i.e., the unit sphere of X does not contain any non-degenerate segment. It is also well-known (see e.g. [2, Chapter II.1] or [3, p. 1782]) that every LUR norm has the *Kadets property*, i.e., the weak and norm topologies agree on its unit sphere.

The space X is said to be *average locally uniformly rotund* (ALUR for short) if for every x in the unit sphere of X and every sequence of Bochner integrable functions $f_n : [0, 1] \longrightarrow B_X$ such that $\lim_n \|x - \int_0^1 f_n(t)\mathrm{d}t\| = 0$ we have $\lim_n \int_0^1 \|x - f_n(t)\|\mathrm{d}t = 0$. ($B_X$ denotes the closed unit ball of X.)

The ALUR property was characterized in [4,5], where it was shown that a Banach space X is ALUR if and only if X is strictly convex and has the Kadets property, if and only if X has the *G property*, i.e., every element of the unit sphere of X is an ϵ-denting point of B_X for each $\epsilon > 0$. Recall that if K is a subset of X, then an element $x \in K$ is said to be an *ϵ-denting point* of K if there is a weak open half-space $H \subset X$ (i.e., a set of the form $H = f^{-1}(a, \infty)$ for some $f \in X^* \setminus \{0\}$ and $a \in \mathbb{R}$) such that $x \in H$ and diam $(H \cap K) < \epsilon$.

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