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Symmetry of integral equation systems with Bessel kernel on bounded domains

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ABSTRACT

In this paper, we investigate the symmetry of integral equation systems with Bessel kernel on bounded domains. Under some natural integrability conditions, we prove that the domains are balls and all positive solutions are radially symmetric and monotonic decreasing.

1. Introduction

In this paper, we study the following system of higher order integral equation on bounded C^1 domain Ω:

$$\begin{cases} u(x) = \displaystyle\int_\Omega G_\alpha(x-y)u^a(y)v^b(y)\mathrm{d}y + C, & x \in \Omega, \\ v(x) = \displaystyle\int_\Omega G_\beta(x-y)u^c(y)v^d(y)\mathrm{d}y + D, & x \in \Omega, \\ u = C_1, v = C_2 \quad \text{on } \partial\Omega, \\ u > 0, v > 0 \quad \text{in } \Omega \end{cases} \tag{1.1}$$

where $a, b, c, d,\ \alpha,\ \beta,\ C_1, C_2, C, D$ are constants satisfying

$$\begin{cases} a, b, c, d \geq 0, 0 < \alpha, \beta < n, \\ C, D, C_1, C_2 > 0, \end{cases} \tag{1.2}$$

and G_α is the Bessel kernel defined as

$$G_\alpha(x) = \frac{1}{\gamma(\alpha)} \int_0^\infty \exp\left(-\frac{\pi|x|^2}{\delta}\right) \exp\left(-\frac{\delta}{4\pi}\right) \delta^{\frac{\alpha-n}{2}} \frac{\mathrm{d}\delta}{\delta}$$

with $\gamma(\alpha) = (4\pi)^{\frac{\alpha}{2}} \Gamma(\frac{\alpha}{2})$.

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