Nonlinear Analysis 74 (2011) 5566–5571

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Removable singularities of solutions of degenerate nonlinear elliptic equations on the boundary of a domain

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ARTICLE INFO

Article history:
Received 25 April 2010
Accepted 18 May 2011
Communicated by Enzo Mitidieri

Keywords:
Removable singularity
Degenerate
Nonlinear
Elliptic
Boundary of a domain

ABSTRACT

The removability of singularities of solutions for the Dirichlet problem for degenerate nonlinear elliptic equations on the boundary of a domain is studied. A method based on a priori energetic estimates of solutions to elliptic boundary value problems is used. The growth in the vicinity of a boundary point (finite or at infinity) for generalized solutions is studied.

1. Introduction

The goal of this paper is to study the removable singularities of solutions for the Dirichlet problem for degenerate nonlinear elliptic equations on the boundary of a domain. For that, a method based on a priori energetic estimates of solutions to elliptic boundary value problems is used. We study the growth in the vicinity of a boundary point (finite or at infinity) for generalized solutions. The method applied differs from that used for obtaining appropriate results in a linear situation.

The corresponding results for linear equations were obtained in the papers of Carleson [1], Kondratyev and Oleynik [2], Oleynik and Iosifyan [3], Kondratyev and Landis [4], Gilbarg and Trudinger [5], Gadjiev and Mamedova [6], Diederich [7], and Harvey and Polking [8], and those for nonlinear equations in the papers of Kilpelainen and Zhong [9] and others. The paper is organized as follows. In Section 2, we present some definitions and auxiliary results. In Section 3, we give the main results for the behaviour of the energy integral. In Section 4 we give the main results for the removability of the singularity.

Let $\Omega \subset R^n$, $n \geq 2$, be a bounded domain. Consider the following equation:

$$\sum_{|\alpha|\leq m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, u, Du, \dots, D^m u) = \sum_{|\alpha|\leq m} (-1)^{|\alpha|} D^\alpha F_\alpha(x), \tag{1}$$

where

$$D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \dots \partial x_n^{\alpha_n}}, \quad |\alpha| = \alpha_1 + \alpha_2 + \cdots + \alpha_n, \ \ m \geq 1.$$

Assume that the coefficients $A_\alpha(x, \xi)$ of Eq. (1) are measurable with respect to $x \in \bar{\Omega}$, are continuous with respect to $\xi \in R^M$ (M is the number of different multi-indices of length no greater than m) and satisfy the conditions

$$\sum_{|\alpha|=m} A_\alpha(x, \xi)\xi_\alpha^m \geq \omega(x) \left|\xi^m\right|^p - c_1\omega(x) \sum_{i=1}^{m-1} |\xi_i|^p - f_1(x),$$

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0362-546X/$ – see front matter
doi:10.1016/j.na.2011.05.040