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Periodic solutions of nonlocal semilinear fourth-order differential equations☆

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ABSTRACT

In this paper we study the existence of periodic solutions with prescribed wavelength for two classes of nonlocal fourth-order nonautonomous differential equations. Existence of nontrivial solutions for the first equation is proved using Clark's theorem. Existence of nontrivial solutions for the second equation is proved using the symmetric mountain-pass theorem.

1. Introduction

Population dynamics of a variety of species in nature have been successfully addressed with the help of reaction–diffusion equations [1]. The interactions among the individuals and with the environment are described by the reaction term (nonlinearity) while the transport is typically modeled by the diffusive term. Interactions with the environment are normally assumed to occur in the immediate outskirts of the individual, with a resulting spatially local representation. Clearly, this may not be realistic in some systems. There may be an interaction-induced modification of the environment around the individual and the elemental reaction term may therefore be more appropriately spatially nonlocal. Pattern formation resulting in such a situation which calls for the use of an integral kernel instead of a local derivative has been analyzed recently [2–4]. Whereas the analysis of patterns given in [2–4] were based on the use of a logistic reaction–diffusion term, our present investigations assume a Nagumo term which provides an additional zero in the nonlinearity relative to the logistic case. The physical content behind such a term is the Allee effect [5].

In paper [6], the author studied the following problem

$$\begin{cases} u^{iv} - pu'' - a(x)u + b(x)u^3 = 0, \quad 0 < x < L, \\ u(0) = u''(0) = u(L) = u''(L) = 0. \end{cases}$$

The case $p > 0$ is referred to as the extended Fisher–Kolmogorov equation while the case $p < 0$ yields the Swift–Hohenberg equation (see [7,8]). Using the variational method, the author established a multiplicity result of existence of periodic solutions with prescribed wavelength for

$$u^{iv} - pu'' - a(x)u + b(x)u^3 = 0.$$

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