Nonlinear Analysis 74 (2011) 2813–2818

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Effective velocity in compressible Navier–Stokes equations with third-order derivatives

Ansgar Jüngel

Institute for Analysis and Scientific Computing, Vienna University of Technology, Wiedner Hauptstr. 8-10, 1040 Wien, Austria

ARTICLE INFO

Article history:
Received 21 April 2010
Accepted 5 January 2011

MSC:
35G25
35Q53
35Q40
76Y05

Keywords:
Korteweg-type models
Quantum hydrodynamic equations
Viscous Euler system
Energy estimates
BD entropy

ABSTRACT

A formulation of certain barotropic compressible Navier–Stokes equations with third-order derivatives as a viscous Euler system is proposed by using an effective velocity variable. The equations model, for instance, viscous Korteweg or quantum Navier–Stokes flows. The formulation in the new variable allows for the derivation of an entropy identity, which is known as the BD (Bresch–Desjardins) entropy equation. As a consequence of this estimate, a new global-in-time existence result for the one-dimensional quantum Navier–Stokes equations with strictly positive particle densities is proved.

1. Introduction

Fluid models with third-order derivatives occur, for instance, in the theory of capillarity with diffuse interfaces [1], in viscous shallow-water models [2], and quantum hydrodynamic equations for semiconductors [3]. In this note, we consider a specific class of third-order derivatives including special viscous Korteweg-type models and quantum Navier–Stokes equations. For these models, it is usually far from being trivial to derive suitable a priori (entropy) estimates and to prove the global-in-time existence of solutions, due to the strongly nonlinear third-order derivatives.

Bresch and Desjardins have found a new mathematical entropy, called BD entropy, for compressible Navier–Stokes models and viscous shallow-water equations allowing for density-dependent viscosities and third-order capillary terms [4,5,2]. The BD entropy estimate is based on the definition of a new velocity variable involving gradients of the particle density. In this note, we show that the BD entropy is a mathematical entropy for a class of Navier–Stokes equations with third-order derivatives, which are *not* covered by the class of equations studied in [6,5,7,2]. Our class of equations contains Korteweg-type fluid models (not included in [2]) and quantum Navier–Stokes equations [8,9].

Our main discovery is that, under suitable assumptions, the third-order expression can be *eliminated* by using the new velocity variable. We notice that the third-order terms do *not* vanish in the formulation of [5,2]. The particle density and the new velocity then solve a viscous Euler system, which allows for the derivation of entropy estimates and a global-in-time existence result in one space dimension.

More precisely, we consider the barotropic Euler equations for the particle density n and the fluid velocity u,

$$n_t + \operatorname{div}(nu) = 0, \tag{1}$$

$$(nu)_t + \operatorname{div}(nu \otimes u) + \nabla p(n) = nf + \operatorname{div}(S + K) \quad \text{in } \mathbb{R}^d,\ t > 0, \tag{2}$$

E-mail address: juengel@anum.tuwien.ac.at.

0362-546X/$ – see front matter
doi:10.1016/j.na.2011.01.002