Nonlinear Analysis 74 (2011) 2468–2474

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Global existence for the higher-order Camassa–Holm shallow water equation

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ARTICLE INFO

Article history:
Received 25 January 2010
Accepted 4 December 2010

Keywords:
Higher-order Camassa–Holm
Local well-posedness
Conservation law
Global existence

ABSTRACT

In this paper, we investigate the global existence of the higher-order Camassa–Holm equation in the case of $k = 2$. We prove the local well-posedness of this equation and find a conservation law. Then a global existence result is obtained.

1. Introduction

The nonlinear dispersive wave equation

$$u_t + 2\omega u_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \tag{1.1}$$

was derived by Camassa and Holm [1] as a model for the unidirectional propagation of shallow water waves over a flat bottom. Here $u(t, x)$ stands for the fluid velocity at time t in the spatial x direction (or equivalently the height of the free surface of water above a float bottom), ω is a constant related to the critical shallow water wave speed. Eq. (1.1) is the well-known Camassa–Holm equation. It has a bi-Hamiltonian structure [2,3] and is completely integrable [1,4]. Moreover, it has many conservation laws (see [5]):

$$E_1 = \int_R u\mathrm{d}x, \qquad E_2 = \int_R (u^2 + u_x^2)\mathrm{d}x, \qquad E_3 = \int_R (u^3 + uu_x^2)\mathrm{d}x.$$

And it also has solitary wave solutions [6–8] to the form $ce^{-|x-ct|}$, $c \in R$, which is called peakon because they have a discontinuous first derivative at the wave peak. In addition to smooth solutions, the author in [6] obtained that there are a multitude of travelling waves with singularities: cuspons, stumpons and composite waves.

The well-posedness of the Camassa–Holm equation has been studied extensively [9–16]. Using a regularization technique, Li and Olver in [13] established the local well-posedness in the Sobolev space H^s with any $s > \frac{3}{2}$ for Eq. (1.1). In the condition of the first derivative of initial value belongs to $L^\infty(R)$; they also obtained an existence theorem for (1.1) in $H^s(R)$, $1 < s \le \frac{3}{2}$. Similar results for the local well-posedness of (1.1) were obtained by Rodriguez-Blanco [17] by applying the Kato theory [18] for the quasilinear equations. The global well-posedness in case $s > \frac{3}{2}$ to the Camassa–Holm equation was also established, provided that $\int |u_0|\, \mathrm{d}x < +\infty$ and $(1 - \partial_x^2)u_0$ does not change sign. Furthermore, the global existence and blow-up for this equation have also been well studied in [10,19,12–16]. Here blow-up means that the slope of the solution becomes unbounded while the solution itself stays bounded. The first studies on the Cauchy problem for the CH

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0362-546X/$ – see front matter © 2011 Published by Elsevier Ltd
doi:10.1016/j.na.2010.12.002