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Nonlinear periodic boundary value problems with sign-changing Green's function☆

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ABSTRACT

We study the existence and nonexistence of positive solutions of nonlinear periodic boundary value problems

$$u'' + a(t)u = \lambda b(t)f(u), \quad a.e.\ t \in [0, T],$$
$$u(0) = u(T), \qquad u'(0) = u'(T),$$

where $b \succ 0$, $a \succ 0$ and the Green's function $G(t, s)$ of the linear problem

$$u'' + a(t)u = 0, \quad a.e.\ t \in [0, T],$$
$$u(0) = u(T), \qquad u'(0) = u'(T)$$

may change sign on $[0, T] \times [0, T]$.

1. Introduction

Let $T > 0$ be given. For $a \in L^p(0, T)$, write $a \succ 0$ if $a \geq 0$ for a.e. $t \in [0, T]$ and it is positive in a set of positive measure; denote as $\|a\|_p$ the norm of a in $L^p(0, T)$. For $\alpha > 0$, let

$$K(\alpha, T) = \begin{cases} \dfrac{2\pi}{\alpha T^{1+2/\alpha}} \left(\dfrac{2}{2+\alpha}\right)^{1-2/\alpha} \left(\dfrac{\Gamma\left(\frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{\alpha}\right)}\right)^2, & 1 \leq \alpha < \infty, \\ 4/T, & \alpha = \infty. \end{cases} \tag{1.1}$$

Torres [1] used some eigenvalue arguments from [2,3] to study the sign of the Green's function $G(t, s)$ of the linear problem

$$u'' + a(t)u = 0, \quad \text{a.e. } t \in (0, T), \tag{1.2}$$

$$u(0) = u(T), \qquad u'(0) = u'(T). \tag{1.3}$$

He proved the following:

Theorem A. *Assume that:*

(A0) $a \in L^p(0, T)$ *for some* $1 \leq p \leq \infty$ *with* $a \succ 0$ *and*

$$\|a\|_p \leq K(2p^*, T). \tag{1.4}$$

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