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Convergence of the projected gradient method for quasiconvex multiobjective optimization

J.Y. Bello Cruz, L.R. Lucambio Pérez *, J.G. Melo

Instituto de Matemática e Estatística, Universidade Federal de Goiás, Campus Samambaia, CEP 74001-970 GO, Goiânia, Brazil

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ABSTRACT

We consider the projected gradient method for solving the problem of finding a Pareto optimum of a quasiconvex multiobjective function. We show convergence of the sequence generated by the algorithm to a stationary point. Furthermore, when the components of the multiobjective function are pseudoconvex, we obtain that the generated sequence converges to a weakly efficient solution.

1. Introduction

In multicriteria optimization, several objective functions have to be minimized simultaneously. Usually, no single point will minimize all given objective functions at once, and so the concept of optimality has to be replaced by the concept of Pareto-optimality or efficiency.

Finding efficient points for the preference order induced by the Paretian cone $\mathbb{R}^m_+$ is a very relevant problem in many areas, such as engineering, statistics, design and others (see [1–7]).

A popular strategy for solving multiobjective optimization problems is the scalarization approach. The most widely used scalarization technique is the weighting method. Basically, one minimizes a linear positive combination of the objectives, where the vector of "weights" is not known a priori and, so, this procedure may lead to unbounded numerical problems, which, therefore, may lack minimizers (see [8–10]). Another disadvantage of this approach is that the choice of the parameters is not known in advance, leaving the modeler and the decision-maker with the burden of choosing them.

The class of quasiconvex multiobjective functions has many applications in real life problems, for example, in economy. For this kind of problems, the weighting method has another weakness: positive combinations of quasiconvex functions may not be quasiconvex.

We are interested in the study of the projected gradient algorithm for solving quasiconvex multiobjective optimization problem. An advantage of this method over the weighting method is that it exploits the structure of quasiconvexity of the problem, as we will show.

Recently, the gradient method for multiobjective optimization problems was proposed in [11]. Since then, it has been considered in more general settings, for instance, for vector optimization problems (see [12]), and for constrained vector

* Corresponding author. Tel.: +55 62 35211424; fax: +55 62 35211208.
E-mail addresses: yunier@impa.br (J.Y. Bello Cruz), lrlp@mat.ufg.br (L.R. Lucambio Pérez), jefferson@mat.ufg.br (J.G. Melo).

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