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Strong convergence theorem for pseudo-contractive mappings in Hilbert spaces

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ABSTRACT

The purpose of this paper is to construct an Ishikawa type of hybrid algorithm for pseudo-contractive mappings in Hilbert spaces. Our results extend the recent ones announced by Yao et al. [Y.H. Yao, Y.C. Liou, G. Marino, A hybrid algorithm for pseudo-contractive mappings, Nonlinear Anal. 71 (2009) 4997–5002] and many others.

1. Introduction

Let H be a real Hilbert space and C be a nonempty closed convex subset of H. Let T be a self-mapping of C. We use $F(T)$ to denote the set of fixed points of T(i.e., $F(T) = \{x \in C : Tx = x\}$).

Definition 1.1 (*[1]*)**.** A mapping $T : C \to C$ is said to be strict pseudo-contraction if there exists a constant $0 \leq k < 1$ such that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + k\|(I - T)x - (I - T)y\|^2, \tag{1.1}$$

for all $x, y \in C$. If $k = 1$, then T is said to be a pseudo-contraction, i.e.,

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + \|(I - T)x - (I - T)y\|^2, \tag{1.2}$$

is equivalent to,

$$\langle (I - T)x - (I - T)y, x - y\rangle \geq 0, \tag{1.3}$$

for all $x, y \in C$.

The class of strict pseudo-contractions extend the class of nonexpansive mapping. (A mapping T is said to be nonexpansive, if $\|Tx - Ty\| \leq \|x - y\|$, for all $x, y \in C$.) That is, T is nonexpansive if and only if T is a 0-strict pseudo-contraction. The pseudo-contractive mapping includes the strict pseudo-contractive mapping.

Iterative methods for finding fixed points of nonexpansive mappings are an important topic in the theory of nonexpansive mappings and have wide applications in a number of applied areas, such as the convex feasibility problem [2–4], the split feasibility problem [5–7] and image recovery and signal processing [8–10] etc. However, the Picard sequence $\{T^n x\}_{n=0}^{\infty}$ often

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