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A triangular map of type 2^∞ with positive topological entropy on a minimal set

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ABSTRACT

We provide a class of triangular maps of the square, $(x, y) \mapsto (f(x), g_x(y))$ of type 2^∞, i.e., such that the periods of periodic points are the powers of 2, which has a minimal set supporting positive topological entropy. This improves the famous example by S. Kolyada from 1992 and contributes to the solution of an old problem by A.N. Sharkovsky.

1. Introduction

Let $I = [0, 1]$ be the unit interval, and $\mathcal{C}$ the class of continuous maps $I \to I$. A *triangular map* is a continuous map $F : I^2 \to I^2$ of the form $F(x, y) = (f(x), g_x(y))$. We denote the class of triangular maps by $\mathcal{T}$. It is a special class of two-dimensional maps, sharing many properties with $\mathcal{C}$. For continuous maps of the interval there is a long list of more than 50 mutually equivalent conditions characterizing maps with zero topological entropy [1]. Some of them are not applicable but more than 30 can be considered for maps in $\mathcal{T}$. It turns out that, e.g., the Sharkovsky's theorem on coexistence of periodic orbits, originally proved for maps in $\mathcal{C}$, remains valid also for the class $\mathcal{T}$. On the other hand, there are essential differences between dynamical properties of $\mathcal{C}$ and $\mathcal{T}$. Many of them can be already illustrated by the maps of *type* 2^∞, i.e., maps such that the periods of periodic points are the powers of 2. These maps in $\mathcal{C}$ — but not in $\mathcal{T}$ — always have zero topological entropy. There is a problem formulated in the eighties by Sharkovsky to find all implications between these conditions in the class $\mathcal{T}$. Partial results can be found, e.g., in [2,3] or [4], or in a survey paper [5]. The following is the famous result by Kolyada from 1992.

Theorem 1 (*Cf. [6]*). *There is a map $F \in \mathcal{T}$ of type 2^∞ with positive topological entropy.*

In 2004, proof of this theorem was simplified [4]. But Kolyada's example can be strengthened. Denote by $h(\varphi)$ the topological entropy of a map φ. Recall that a point x is *uniformly recurrent* or, $x \in UR(\varphi)$ if, for any neighborhood U of x, the set of times n such that $\varphi^n(x) \in U$, has bounded gaps. Equivalently, $x \in UR(\varphi)$ if x is a *minimal point* of φ, i.e., the

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