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Stably average shadowable homoclinic classes

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ABSTRACT

Let p be a hyperbolic periodic saddle of a diffeomorphism of f on a closed smooth manifold M, and let $H_f(p)$ be the homoclinic class of f containing p. In this paper, we show that if $H_f(p)$ is locally maximal and every hyperbolic periodic point in $H_f(p)$ is uniformly far away from being nonhyperbolic, and $H_f(p)$ has the average shadowing property, then $H_f(p)$ is hyperbolic.

1. Introduction

It has been a main aim, as regards differentiable dynamical systems, for the last few decades, to understand the influence of a robust dynamic property on the behavior of the tangent map of the system. For instance, Mañé [3] proved that any robustly transitive diffeomorphism f of a closed surface S is an Anosov diffeomorphism. To study this problem, some people try to understand the influence of a robust dynamic property in systems with some shadowing properties, since these shadowing properties are closely related to the stability of systems (see [1,2,4–7]).

In this paper, we introduce the notion of the C^1-stably average shadowing property, and study the case where the homoclinic class has the stably average shadowing property.

Let us pass to the main definitions and results. Let M be a closed C^∞ Riemannian manifold. Denote by d the distance on M induced from a Riemannian metric $\|\cdot\|$ on the tangent bundle TM. Let $\mathrm{Diff}(M)$ be the space of diffeomorphisms of M endowed with the C^1-topology. Let $f : M \to M$ be a diffeomorphism. For $\delta > 0$, a sequence of points $\{x_i\}_{i=a}^{b}$ $(-\infty \le a < b \le \infty)$ in M is called a *δ-pseudo-orbit* of f if

$$d(f(x_i), x_{i+1}) < \delta$$

for all $a \le i \le b-1$. For $\delta > 0$ a sequence $\{x_i\}_{i=-\infty}^{\infty}$ in M is called a *δ-average pseudo-orbit* of $f \in \mathrm{Diff}(M)$ if there is a natural number $N = N(\delta) > 0$ such that for all $n \ge N$, and $k \in \mathbb{Z}$,

$$\frac{1}{n}\sum_{i=1}^{n} d(f(x_{i+k}), x_{i+k+1}) < \delta.$$

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