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Characterization of a condition related to a class of preinvex functions

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ABSTRACT

In the theory of invex functions, a very important role is played by the so-called Condition C. We present a convenient characterization of this condition in terms of starlike sets and projections.

1. Introduction

It is well known that in the theory of mathematical programming, a central role is played by convex functions and some of their generalizations. One of such generalizations is the notion of invexity introduced by Hanson [1]. Let us recall the definition of an invex set (see e.g. [2]). We say that a nonempty subset S of $\mathbb{R}^n$ is *invex with respect to a function* $\eta : S \times S \to \mathbb{R}^n$, provided

$$u + \lambda\eta(x, u) \in S \quad \text{for } x, u \in S,\ \lambda \in [0, 1].$$

Furthermore, if a set S is invex with respect to η, then a function $f : S \to \mathbb{R}$ is said to be *preinvex* (with respect to η) if

$$f(u + \lambda\eta(x, u)) \leq \lambda f(x) + (1 - \lambda)f(u) \quad \text{for } x, u \in S,\ \lambda \in [0, 1].$$

Interesting results concerning invex sets and preinvex functions can be found, e.g. in [3–5,2,1,6–15]. The notions of invex set and preinvex function are quite general. Therefore, to get some deeper results, usually, one has to assume additional conditions concerning η. One of the frequently applied conditions is the so-called *Condition C*, introduced by Mohan and Neogy [7]. A function $\eta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ satisfies Condition C, provided, for every $x, y \in \mathbb{R}^n$ and $\lambda \in [0, 1]$, the following two equalities hold:

$$\eta(x, y + \lambda\eta(x, y)) = (1 - \lambda)\eta(x, y),$$
$$\eta(y, y + \lambda\eta(x, y)) = -\lambda\eta(x, y).$$

For several interesting results concerning Condition C we refer, e.g. to [3,4,6,7,13–15]. The main purpose of the present paper is to give a convenient characterization of functions satisfying Condition C. Note that, in fact, the definitions of invex set, preinvex function and Condition C, do not require the structure of $\mathbb{R}^n$. Thus, they could be formulated in a more general setting, where $\mathbb{R}^n$ is replaced by an arbitrary real vector space X. In order to present the main result of the paper, we need to recall the definition of a starlike set (see e.g. [16,17]). A nonempty subset S of X is called *starlike with respect to the point*

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