Nonlinear Analysis 74 (2011) 1286–1289

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

The existence of solutions to certain quasilinear elliptic equations

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ARTICLE INFO

Article history:
Received 23 June 2009
Accepted 1 October 2010

MSC:
primary 35D05
35J25
secondary 46E35

Keywords:
Quasilinear elliptic equation
$W^{2,p}$-estimate
$W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$ solution

ABSTRACT

Let $Lu = -\sum_{i,j=1}^{N} a_{ij}(x, u)D_{ij}u + c(x, u)u$. Consider the quasilinear elliptic equation $Lu = f(x, u, \nabla u)$ on a bounded smooth domain Ω in $\mathbb{R}^N$, where $c(x, r) \geq \alpha > 0$, $f(x, r, \xi) = o[|r| + h(|r|)|\xi|^2]$. It is shown that if the oscillation of $a_{ij}(x, r)$ with respect to r is sufficiently small, then there exists a solution $u \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$ to the equation $Lu = f(x, u, \nabla u)$.

1. Introduction

Let Ω be a *bounded* $C^{1,1}$ domain in $\mathbb{R}^N$, $N \geqslant 3$, and let L_v, L, D_v, D be *elliptic operators* defined by

$$L_v u = -\sum_{i,j=1}^{N} a_{ij}(x, v)D_{ij}u + c(x, v)u,$$
$$Lu = L_u u,$$
$$D_v u = -\sum_{i,j=1}^{N} D_i(a_{ij}(x, v)D_j u) + c(x, v)u,$$
$$Du = D_u u,$$

where the coefficients a_{ij} and their derivatives $D_i a_{ij}$, $D_r a_{ij}$ are *bounded* Carathéodory functions, $\sum_{i,j=1}^{N} a_{ij}\xi_i\xi_j \geq \lambda|\xi|^2$, for some constant λ. We shall omit the summation notation $\sum_{i,j=1}^{N}$ and use C for a *generic constant*.

Let $f(x, r, \xi)$ be a locally bounded Carathéodory function defined on $\Omega \times \mathbb{R} \times \mathbb{R}^N$. Consider the quasilinear elliptic equations

$$Lu = f(x, u, \nabla u) \tag{1}$$

in Ω, where $c(x, r) \geq \alpha > 0$ is a bounded Carathéodory function. The present paper aims to investigate the existence of $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$ solutions to (1). For simplicity, we denote in the sequel $s = (r, \xi)$, $|s| = |r| + |\xi|$, $f(x, r, \xi) =$

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doi:10.1016/j.na.2010.10.001