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A best proximity point theorem for weakly contractive non-self-mappings

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ABSTRACT

Let us consider a map $T : A \to B$, where A and B are two nonempty subsets of a metric space X. The aim of this article is to provide sufficient conditions for the existence of a unique point x^* in A, called the best proximity point, which satisfies $d(x^*, Tx^*) = \operatorname{dist}(A, B) := \inf\{d(a, b) : a \in A, b \in B\}$. Our result generalizes a result due to Rhoades [B.E. Rhoades, Some theorems on weakly contractive maps, Nonlinear Analysis TMA, 47(2001), 2683–2693] and hence it provides an extension of Banach's contraction principle to the case of non-self-mappings.

1. Introduction

Let A and B be nonempty subsets of a metric space X. A map $T : A \to B$ is said to be a k-contraction map if there is a constant $k \in [0, 1)$ such that $d(Tx, Ty) \le kd(x, y)$, for all $x, y \in A$. The well-known Banach contraction principle states that if A is a complete subset of X and T is a contraction map which maps A into itself, then T has a unique fixed point in A. Due to its wide applications in various fields, a huge number of generalizations and extended versions of this principle appear in the literature. For some important and interesting generalizations of Banach's principle, one can refer to [1,2].

The following notion of a weakly contractive self-map was introduced by Alber and Guerre in [3].

Definition 1 (*[3]*). Let X be a metric space and A be a nonempty subset of X. A map $T : A \to A$ is said to be a weakly contractive self-map if

$$d(Tx, Ty) \le d(x, y) - \psi(d(x, y)), \quad \text{for all } x, y \in A,$$

where $\psi : [0, \infty) \to [0, \infty)$ is a continuous and nondecreasing function such that ψ is positive on $(0, \infty)$, $\psi(0) = 0$ and $\lim_{t\to\infty} \phi(t) = \infty$. If A is bounded, then the infinity condition can be omitted.

In [3], Alber and Guerre proved that if $T : \Omega \to \Omega$ is a weakly contractive self-map, where Ω is a closed convex subset of a Hilbert space, then T has a unique fixed point in Ω. Later, in [4], Rhoades proved that the existence of a unique fixed

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