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A generalization of Mizoguchi and Takahashi's theorem for single-valued mappings in partially ordered metric spaces

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ABSTRACT

We present a generalization of Mizoguchi and Takahashi's fixed point theorem for single-valued mappings in partially ordered metric spaces. As an application of the main result, we give an existence and uniqueness theorem for the solution of a periodic boundary value problem.

1. Introduction

The fixed point theory for multi-valued mappings developed rapidly after the publication of Nadler's paper [1] in which he established a multi-valued version of Banach's contraction principle.

The existence of fixed points in partially ordered metric spaces was first investigated in 1986 by Turinici [2]. Further results in this direction were proved in, e.g., [3–8].

In this paper, we extend the results of Amini-Harandi and O'Regan [9] to ordered metric spaces for single-valued mappings.

2. Preliminaries

Let (X, d) be a metric space. For $x \in X$ and $A \subseteq X$, $d(x, A) = \inf\{d(x, a); a \in A\}$. We denote by $CB(X)$ the class of nonempty bounded subsets of X, and by $K(X)$ the class of all nonempty compact subsets of X. Let H be the Hausdorff metric on $CB(X)$ generated by metric d, that is,

$$H(A, B) = \max\{\sup_{x \in A} d(x, B), \sup_{y \in B} d(y, A)\}$$

for every $A, B \in CB(X)$. A point $p \in X$ is said to be a fixed point of $T : X \to CB(X)$ if $p \in Tp$.

Reich [10] proved that if (X, d) is a complete metric space and $T : X \to CB(X)$ satisfies

$$H(Tx, Ty) \le \alpha(d(x, y))d(x, y)$$

for each $x, y \in X$, where α is a function from $[0, \infty)$ to $[0, 1)$ such that $\limsup_{r \to t^+} \alpha(r) < 1$ for each $t \in (0, \infty)$, then T has a fixed point. Reich raised the question of whether $K(X)$ can be replaced by $CB(X)$ in this result. In [11] Mizoguchi and Takahashi gave a positive answer to the conjecture of Reich; more precisely they proved:

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