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The existence of periodic solutions of non-autonomous second-order Hamiltonian systems☆

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ABSTRACT

The purpose of this paper is to study the existence of periodic solutions for a class of non-autonomous second-order Hamiltonian systems. Some new existence theorems are obtained by using the least action principle and the saddle point theorem.

1. Introduction and the main results

Consider the non-autonomous second-order Hamiltonian system

$$\begin{cases} \ddot{u}(t) = \nabla F(t, u(t)) \\ u(0) - u(T) = \dot{u}(0) - \dot{u}(T) = 0, \end{cases} \tag{1.1}$$

where $u(t) \in R^N$ and $F : [0, T] \times R^N \to R^1$ satisfies the following assumption:

(A) $F(t, x)$ is measurable in t for each $x \in R^N$ and continuously differentiable in x for a.e. $t \in [0, T]$, and there exist $a \in C(R^+, R^+)$ and $b \in L^1(0, T; R^+)$ such that

$$|F(t, x)| + |\nabla F(t, x)| \le a(|x|)b(t)$$

for all $x \in R^N$ and a.e. $t \in [0, T]$.

Here and hereafter, we denote by $\langle \cdot, \cdot \rangle$ and $|\cdot|$ the inner product and norm of R^n respectively. Let H_T^1 be the usual Sobolev space with the norm

$$\|u\|_{H_T^1} = \left(\int_0^T |u(t)|^2 \mathrm{d}t + \int_0^T |\dot{u}(t)|^2 \mathrm{d}t \right)^{\frac{1}{2}}.$$

The functional φ on H_T^1 corresponding to (1.1) is given by

$$\varphi(u) = \int_0^T \left(\frac{1}{2} |\dot{u}(t)|^2 + F(t, u(t)) \right) \mathrm{d}t.$$

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