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A note on the split common fixed-point problem for quasi-nonexpansive operators

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ABSTRACT

Based on the very recent work by Censor and Segal (2009) [1], and inspired by Xu (2006) [9], Zhao and Yang (2005) [10], and Bauschke and Combettes (2001) [2], we introduce and analyze an algorithm for solving the split common fixed-point problem for the wide class of quasi-nonexpansive operators in Hilbert spaces. Our results improve and develop previously discussed feasibility problems and related algorithms.

1. Introduction and preliminaries

Throughout, H is a Hilbert space, $\langle \cdot, \cdot \rangle$ denotes the inner product, and $\| \cdot \|$ stands for the corresponding norm. The split common fixed-point problem (SCFP) is a generalization of the split feasibility problem (SFP) and the convex feasibility problem (CFP); see [1]. In this paper we introduce and study the convergence properties of a relaxed algorithm for solving the SCFP for the class of quasi-nonexpansive operators T such that $I - T$ is closed at the origin. This general class, which properly includes the class of directed operators considered in [2,1], is more desirable, for example, in fixed-point methods in image recovery where, in many cases, it is possible to map the set of images possessing a certain property to the fixed-point set of a nonlinear quasi-nonexpansive operator. Also, for the hybrid steepest descent method, see Yamada [3,4], which is an algorithmic solution to the variational inequality problem over the fixed-point set of certain quasi-nonexpansive mappings and is applicable to a broad range of convexly constrained nonlinear inverse problems in real Hilbert spaces. See also Maingé [5], which proved the convergence of a viscosity algorithm for such a class of operators.

Our main purpose here is to give an extension of the unified framework developed in [1] to quasi-nonexpansive operators by proposing a weak convergence result of the algorithm we will introduce. This will be done in the context of general Hilbert spaces.

To begin with, let us recall that the split feasibility problem (SFP) is to find a point

$$x \in C \quad \text{such that } Ax \in Q, \tag{1.1}$$

where C is a closed convex subset of a Hilbert space H_1, Q is a closed convex subset of a Hilbert space H_2, and $A : H_1 \to H_2$ is a bounded linear operator.

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