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Lipschitz constants for iterates of mean lipschitzian mappings

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ABSTRACT

We present a sharp evaluation of the Lipschitz constant for every iterate for mean lipschitzian mappings, solving a problem posed by K. Goebel and M. A. Japón Pineda.

1. Introduction

Let (M, ρ) be a metric space. We say that a mapping $T : M \to M$ is *lipschitzian* if there is a constant $k \geq 0$ such that for any $x, y \in M$,

$$\rho(Tx, Ty) \leq k\rho(x, y).$$

If k is the minimum number which satisfies this condition, we say that T has Lipschitz constant k; we denote it by $k_\rho(T)$, or simply $k(T)$ if in the context it is clear in which metric we are working. Lipschitzian mappings with $k = 1$ are called *nonexpansive*. If $k < 1$, T is called a *contraction*.

For any two lipschitzian mappings $T, S : M \to M$, we have the following evaluation:

$$k(T \circ S) \leq k(T)k(S).$$

This condition regulates the growth of the Lipschitz constants of iterates of a mapping T as we can see:

$$k(T^{m+n}) \leq k(T^m)k(T^n)$$

and consequently

$$k(T^n) \leq k(T)^n.$$

In [1] Goebel and Japón Pineda and in [2] Goebel and Sims defined a generalization of the Lipschitz condition: suppose that $\alpha = (\alpha_1, \ldots, \alpha_n)$, where $\alpha_1, \ldots, \alpha_n \in \mathbb{R}$, $\alpha_i \geq 0$, $\alpha_1 > 0$, $\alpha_n > 0$ and $\sum_{j=1}^n \alpha_j = 1$; we say that T is an α-lipschitzian mapping for the constant k if for each $x, y \in M$

$$\sum_{j=1}^{n} \alpha_j \rho(T^j x, T^j y) \leq k\rho(x, y). \tag{1.1}$$

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