International Journal of Mechanical Sciences 53 (2011) 617–627

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Scattering of flexural wave in a thin plate with multiple circular inclusions by using the multipole method

W.M. Lee [a,*], J.T. Chen [b]

[a] Department of Mechanical Engineering, China University of Science and Technology, Taipei, Taiwan
[b] Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan

ARTICLE INFO

Article history:
Received 13 June 2010
Received in revised form
18 May 2011
Accepted 19 May 2011

Keywords:
Scattering
Circular inclusion
Flexural wave
Dynamic moment concentration factor
Far-field scattering pattern
Addition theorem

ABSTRACT

The multipole method is presented to analytically solve the scattering of flexural wave by multiple circular inclusions in an infinite thin plate. The near-field dynamic moment concentration factor (DMCF) and the far-field scattering pattern are both investigated in this paper. The former has a connection with the fatigue failures and the defects in plate-like structures can be detected by the latter. Owing to the addition theorem, the multipole expansion for the multiple scattering fields can be transformed into one coordinate system centered at one circle where continuity conditions are required. In this way, a coupled infinite linear algebraic system is derived as an analytical model for an infinite thin plate with multiple circular inclusions subject to an incident flexural wave. The convergence analysis is conducted to provide the guideline of usage for the proposed method. The effects of the size and thickness of the flexible inclusion, and the central distance between inclusions on the near-field DMCF and the far-field scattering pattern are investigated in the numerical experiments. It shows that the scattering pattern correlates closely with the size and thickness of weakness, indicating the importance of the scattering pattern to detect the various defects. In addition, the DMCF of two corrosion defects is larger than that of one. Therefore, it is essential to evaluate structural safety when multiple circular defects are very close to each other. The effect of the space between the inclusions on the near-field DMCF is different from that on the far-field scattering pattern.

1. Introduction

Thin plates with multiple circular inclusions are commonly observed in many practical engineering structures. These inclusions, or inhomogeneities, usually take place in terms of either the thickness reduction due to corrosion in a metallic plate or the strength degradation caused by delamination in a quasi-isotropic composite plate. The other examples can be found in the plates with bolts or rivets, which are often used in the engineering structure. The deformation and corresponding stresses induced by dynamic loading are propagated throughout the structure by means of wave. At the near field of inclusion (or defect), flexural wave scattered in all directions recursively interacts with the incident wave. It turns out that the scattering of the stress wave induces dynamic stress concentration [1], which results in fatigue failure and reduces the loading capacity. On the other hand, the far-field scattering pattern can determine the size and severity of structural damage in plate-like structure by using a quantitative in situ structural health-monitoring system, one of the non-destructive inspections.

One of the early research studies in the analytical approach to the dynamic stress concentrations is that of Nishimura and Jimbo [2]. The stresses in the vicinity of a spherical inclusion in the elastic solid under a harmonic force were investigated. Pao [3] studied the scattering of flexural waves and dynamic stress concentrations around a circular hole, and proposed an analytical solution. Thau and Lu [4] studied the dynamic stress concentration at a cylindrical inclusion in an elastic medium. Since then, most research work has focused on the scattering of elastic wave and the resulted dynamic stress concentration, and has led to a rapid development of analytical or numerical approach such as the method of wave function expansion, the complex variable method, the boundary integral equation method and the boundary element method [1].

Norris and Vemula [5] considered the scattering of flexural waves by circular inclusions with different plate properties and obtained numerical results. Squire and Dixon [6] applied the wave function expansion method to study the scattering properties of a single coated cylindrical anomaly located in a thin plate on which flexural waves propagate. Wang and Chang [7] presented a theoretical and experimental investigation of the scattering

* Corresponding author. Tel.: +886 2 27867048; fax: +886 2 27867253.
E-mail addresses: wmlee@cc.cust.edu.tw (W.M. Lee), jtchen@mail.ntou.edu.tw (J.T. Chen).

0020-7403/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijmecsci.2011.05.008