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Deformation and stability of an elastica under a point force and constrained by a flat surface

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ABSTRACT

This paper studies the deformation and stability of a pinned elastica under a point force moving quasi-statically from one end to the other. The elastica is constrained by a rigid plane wall containing the two ends. Three types of equilibrium configurations can be found; they are non-contact, one-point contact, and one-line contact on the side. A vibration method is adopted to determine the stability of the calculated deformations. In order to take into account the variation of the contact region between the elastica and the plane wall during vibration, an Eulerian version of the governing equations is adopted. It is found that all the point-contact deformations are unstable. On the other hand, there are two different mechanisms a line-contact deformation becomes unstable; one through a secondary buckling and the other through a limit-point bifurcation. In the secondary buckling, the length of the line-contact segment and the axial force satisfy the Euler buckling criteria for a pinned-clamped column. On the other hand, when a line-contact deformation becomes unstable via a limit-point bifurcation, the axial force does not exceed the Euler buckling load. The theoretical predictions are confirmed by experimental observations.

1. Introduction

In this paper we investigate the behavior of a plane elastica restrained by hinges on both ends and subject to a point force moving quasi-statically from one end to the other. The lateral deformation of the elastica is constrained by a rigid wall on the base plane containing the two ends. This mechanics problem arises when a vehicle travels on a blown-up highway pavement, which may be caused by temperature elevation [1]. The elastica and the rigid surface represent the blown-up pavement and the foundation of the highway. The rigid wall acts as a unilateral constraint because it can sustain compression but not tension when the elastica touches the foundation.

The interaction between an elastica and a rigid surface attracts intensive research interests because of its wide practical applications. Wang [8,9,10] investigated extensively the behavior of a heavy elastica, finite or infinitely long, on a rigid foundation. Plaut and Mróz [2] studied the effects of initial curvature and load eccentricity on the deformation of a pinned heavy elastica on a rigid surface. Kublanov and Bottega [3] calculated the deformation of a thin buckled film on a flat substrate under a point force at the tip. In these works only symmetric deformations were considered. Domokos et al. [4] investigated the symmetry-breaking bifurcation of an infinitely long heavy elastica on a flat surface. Santillan et al. [5] considered the case when the rigid surface is inclined. In all these works, the external load is either uniformly distributed or concentrated at the midpoint. The first purpose of this paper is to study the deformations of the elastica when the point force is off-center.

The emphasis of the previous works was placed on the static deformation of an elastica on a rigid surface. Very often, multiple equilibria under a specified set of loading condition are possible. In order to determine whether a calculated deformation exists in reality, a stability analysis is needed. There is, however, no workable approach available to determine the stability of the elastica when it interacts with the rigid surface. As a consequence, experimental observation was an important tool to validate the existence of a calculated equilibrium in previous researches. The second purpose of this paper is to present a theoretical approach which is capable of predicting the stability of an elastica in contact with the rigid foundation.

In the first part of this paper we discuss the deformation patterns when the point force of various magnitudes travels through the pinned elastica. As expected, multiple equilibrium configurations may co-exist for a specified set of loading and geometric parameters. The second part of this paper focuses on a vibration method which is capable of determining the stability of these equilibrium configurations. For an elastica deformation without contacting the flat surface, the procedure to find its natural frequencies is straightforward and can be found in previous works, for instance, see Refs. [6,5]. The main challenge of the vibration analysis when the elastica touches the rigid surface is that during vibration the contact region may vary from instance to instance. In order to capture this physical essence, an Eulerian description,

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