



# Shear deformable doubly- and mono-symmetric composite I-beams

Nam-Il Kim \*

Department of Civil and Environmental Engineering, Hanyang University, 17 Haengdang-dong, Seoungdong-gu, Seoul 133-791, South Korea

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## ABSTRACT

A shear deformable beam element is developed for the coupled flexural and torsional analyses of thin-walled composite I-beams with doubly- and mono-symmetric cross-sections. The present element includes the transverse shear and the restrained warping induced shear deformation by using the first-order shear deformation beam theory. Governing equations and force–displacement relations are derived from the principle of minimum total potential energy. Then the explicit expressions for displacement parameters are derived by applying the power series expansions of displacement components to simultaneous ordinary differential equations. Finally, the element stiffness matrix is determined using the force–displacement relations. In order to verify the accuracy and the superiority of the beam element developed herein, the numerical solutions are presented and compared with the results obtained from the isoparametric beam elements based on the Lagrangian interpolation polynomial, the detailed three-dimensional analysis results using the shell elements of ABAQUS, and the solutions by other researchers.

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## 1. Introduction

The thin-walled composite I-beams have been used extensively in civil, marine and mechanical engineering as well as in aerospace engineering. These structural components made of advanced composite materials are ideal for structural applications because of the high strength-to-weight and stiffness-to-weight ratios and their ability to be tailored to meet the design requirements of stiffness and strength. These beams might be subjected to bending and twisting types of loadings when used in above applications. Therefore, it becomes important to evaluate exactly their responses under these types of loadings. The first work with the contour-based cross-sectional analysis methods used in this study was conducted by Vlasov [1]. Gjelsvik [2] presented an isotropic beam theory identified with plate segments of the beam. The plate displacements were related to the generalized beam displacements through geometric consideration.

Up to the present, for the static analysis of composite beams, the finite element method has been widely used because of its versatility and accordingly a large amount of work [3–9] was devoted to the improvement of composite finite elements. Back and Will [3] developed the first-order shear deformable beam theory and three different types of finite elements, namely, linear, quadratic and cubic elements to solve the governing equations. In their study, they used reduced integration to alleviate shear locking. Lee [4] developed an analytical model to study the flexural behavior of thin-walled composite beam with doubly symmetric I-section

subjected to uniformly distributed vertical load. He presented the generalized displacements as a linear combination of the one-dimensional Lagrangian interpolation function. Also the two-noded  $C^1$  finite element of 8 DOF per node was developed by Subramanian [5] based on the higher order shear deformation theory for flexural analysis of symmetric laminated composite beams assuming a parabolic variation of transverse shear stress through the thickness of beams. Wu and Sun [6] derived the general finite element with 10 DOF per node for the thin-walled laminated composite beams by applying the zero hoop strain assumption. Maddur and Chaturvedi [7] presented a modified Vlasov-type first-order shear deformation theory which can account for shear deformations of open profile sections. And they simplified their theory for I-beams [8] as a special case and analyzed the deformation response of I-sections made of orthotropic laminated composites based on finite element procedure by using Lagrange interpolation function for the geometric coordinate variables and Hermitian interpolation function for the unknown functions. Shi et al. [9] investigated the influence of the interpolation order of bending strains on the solution accuracy of composite beam element based on HSDT and presented a simple but accurate third-order composite beam element using the assumed strain finite element method.

The boundary element method (BEM), as an effective approach solving the static problem of composite beams, has been used and some works [10–12] was devoted to the improvement of beam element in order to obtain the acceptable results. Mokos and Sapountzakis [10] developed the BEM for the solution of the general transverse shear loading problem of composite beams of arbitrary cross-section. In their study, a stress function was introduced, which fulfilled the equilibrium and compatibility equations and from which

\* Tel.: +82 31 290 7544; fax: +82 31 290 7548.

E-mail address: kni8501@gmail.com