International Journal of Mechanical Sciences 53 (2011) 989–999

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Rolling cylinder on an elastic half-plane with harmonic oscillations in normal force and rotational speed. Part I: Solution of the partial slip contact problem

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ARTICLE INFO

Article history:
Received 9 August 2010
Received in revised form
29 July 2011
Accepted 3 August 2011
Available online 11 August 2011

Keywords:
Rolling contact
Partial slip
Contact mechanics

ABSTRACT

We study the effect of harmonic oscillations during the steady rolling of a cylinder on a plane in partial slip contact conditions in the limit of small oscillations. The solution is an extension of that given in Barber et al. [1] for infinitely large coefficient of friction. Here, the effect of varying normal load and hence contact area is investigated in detail by analyzing the first order variation of the tangential force and the corresponding relative displacements.

In particular, the solution is given in terms of an explicit length scale d in the Flamant solution used as a Green function. Appropriate choice of values of d allows to treat both two-dimensional problems and three-dimensional ones having elliptical contact area sufficiently elongated in the direction of the rotation axis.

Also, this analysis can be used as starting point for corrugation calculations in railway tracks, where oscillations in time of the normal forces can result in non-uniform wear and hence in amplification of the corrugation.

1. Introduction

The rolling of cylinders is a classical problem in mechanics. In practice however the cross-section of the roller is never perfectly circular and equivalently, the countersurface on which the motion occurs is never perfectly flat. Here, we shall look at this problem by perturbing a classical solution due to Carter [2] which deals with the traction (or braking) of a cylindrical wheel. His investigation was originally intended to shed light on rail–wheel wear in locomotives but can be applied in other cases, e.g. rolling at the nano-scale [3], to roll-to-roll printing even in modern nanoimprint lithography [4]. However we shall primarily look at the case of rolling for the classical tractive wheel application.

Recently, an elasticity solution for the two-dimensional problem of a rolling cylinder with applied loads having small sinusoidal oscillations superposed to a mean value has been proposed by Barber and Ciavarella [5], Barber et al. [1] and Afferrante [6]. In the first two papers, the full-stick approximation, which corresponds to assume infinitely large friction coefficient, has been adopted to simplify the problem. In particular, in Barber and Ciavarella [5], the transient effects of rolling has been analytically examined using a perturbation technique on the Winkler model in which the surface displacements of the contacting bodies are assumed simply *proportional to the local tractions*. In Afferrante [6], the Winkler model has been extended to finite values of friction coefficient, implying a finite slip zone in the contact region.

The linear perturbation analysis, as explained in Barber et al. [1], implies that the mean contact area semi-width a_0 in the direction of rolling needs to be sufficiently smaller than the wavelength λ of the initial perturbation. In fact Kalker [7,8] has shown that the time for which the system maintains its 'memory' is equal to the time necessary for a point to move from the leading edge to the trailing edge of the contact area.

One important application of models based on perturbation techniques is in the context of studies of corrugation, where there is a sinusoidal forcing in the form of a corrugated profile over which the rolling takes place, and hence there are oscillations of the displacement and the rolling velocity (creepage). Other authors [9–13] have developed similar approaches to study the phenomenon of the corrugation in railway tracks, which results from an unstable interaction between the dynamics of the vehicle and track and the wear mechanism.

Here, we aim to extend the perturbation analysis to a two-dimensional continuum model of a rolling cylinder on a plane

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doi:10.1016/j.ijmecsci.2011.08.004