International Journal of Mechanical Sciences 53 (2011) 254–261

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

On the stress concentration around a hole in an infinite plate subject to a uniform load at infinity

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ARTICLE INFO

Article history:
Received 12 January 2010
Received in revised form
28 December 2010
Accepted 20 January 2011
Available online 31 January 2011

Keywords:
Plane elasticity
Muskhelishvili method
Stress concentration
Conformal mapping
Schwartz–Chistoffel mapping

ABSTRACT

In this paper stress concentration around a hole in an infinite plate that is subjected to a uniform load at infinity is considered. The stress is calculated by using a modified Muskhelishvili complex variable method. The method is illustrated by several examples of stress distribution around polygonal holes of a complex geometry utilizing the Schwartz–Chistoffel mapping function.

1. Introduction

As it is well known, one of the most powerful methods for the analytical calculation of stress distribution and in particular stress concentration factors around holes in two-dimensional elastic systems is the Muskhelishvili conformal mapping complex variable method. The method is presented in detail in Muskhelishvili's monograph [8] as well as in classical books on the theory of elasticity [3,5,7]. Some further applications and references for contemporary application of the method are given in [1,4,6,9]. When the conformal mapping function is relatively simple, for example of elliptical holes or regular polygonal holes with rounded corners, the method allows one to obtain an elegant closed form solution for stress concentration problems around holes [5,10]. For more complex geometries, the method becomes semi-analytical because one must use numerical methods for obtaining coefficients of power series expansion of mapping functions and calculation of coefficients of power series expansion of complex stress functions [10,11]. The algorithm for semi-analytical calculation of stress distribution using the method is straightforward [11]; however numerical difficulties may arise when the number of coefficients of the mapping function is large (see end of Section 3). Also it seems that the contemporary advances in numerical conformal mapping [2] have yet to be applied to two-dimensional elastic systems.

The purpose of this paper is thus to present a modified Muskhelishvili method by which one may efficiently calculate the stress distribution around holes of relatively complex shapes in infinite plates subjected to a uniform load at infinity. In the Section 1 of the paper the problem is stated; its general solution is given in Section 2. In Section 3 the use of the Schwartz–Chistoffel mapping function in the calculation of stress distribution around polygonal holes is discussed in detail. Numerical examples are then offered before conclusions are presented.

2. General consideration

The object of consideration is an infinite plate with a hole subjected to a uniform stress state at infinity (Fig. 1). On the complex z plane the plate occupies the infinite region S bounded by contour L. Let $\omega(\zeta)$ be the function of complex variable ζ, which maps the exterior of the unit circle $\Sigma: |\zeta| \geq 1$ onto the region S. It is assumed that the mapping function $\omega(\zeta)$ has the following expansion:

$$\omega(\zeta) = Re^{i\delta}\omega_0(\zeta) = Re^{i\delta}\left(\zeta + c_0 + \frac{c_1}{\zeta} + \frac{c_2}{\zeta^2} + \cdots\right)(R > 0, \quad \zeta = \rho e^{i\vartheta}) \tag{1}$$

where R is a real constant, which affects the scale of the hole, and δ is the angle by which the hole must be rotated from its original position.

According to the Kolossov–Muskhelishvili theory [8] the elastic state of the plate is determined by two complex stress functions, $\varphi(\zeta)$ and $\psi(\zeta)$, which are analytic, on the region $|\zeta| > 1$. When they are known, the stress components are calculated by the following stress

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doi:10.1016/j.ijmecsci.2011.01.006