International Journal of Mechanical Sciences 53 (2011) 526–535

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Approximate elastic stress estimates for elbows under internal pressure

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ARTICLE INFO

Article history:
Received 14 October 2010
Received in revised form
23 April 2011
Accepted 27 April 2011
Available online 7 May 2011

Keywords:
Elastic stresses
Elbow
Internal pressure
Finite element analysis

ABSTRACT

This paper presents closed-form approximations to elastic stresses in thick- and thin-walled elbows with attached straight pipes under internal pressure, based on three-dimensional elastic finite element analysis. Elastic stresses in the centre of an elbow are found to be close to an existing closed-form solution, suggesting that the hoop stress varies with the longitudinal position. The FE results indicate that the hoop stress varies linearly with the longitudinal position. Moreover, stresses in the junction of an elbow and straight pipe are shown to be the average of those in the centre of the elbow and in the straight pipe.

1. Introduction

An elbow is commonly used in industrial plants to increase piping flexibility. For design and structural integrity assessment, information on elastic stresses and limit loads for an elbow is needed. Elastic stresses are needed for fatigue design and to estimate stress intensity factors for fracture mechanics analysis. Limit loads, on the other hand, can provide maximum load-carrying capacity of components. For thin-walled elbows under internal pressure, an analytical elastic stress solution was derived by Goodall [1] (see also Miller [2]), from which a limit pressure solution was derived. Regarding limit pressures, the authors and co-workers [3–7] recently performed extensive three-dimensional (3-D) finite element (FE) limit analyses based on an elastic-perfectly plastic material to refine the analytical limit pressure solution proposed by Goodall [1]. It was found [3,4] that FE limit pressures were always higher (by a large amount depending on the pipe and elbow geometries) than the analytical solution. The reason for such differences is stress redistribution in the longitudinal direction [4,6]. For elastic stresses, on the other hand, no essential studies have been done since Goodall's work [1]. Recently, Marie et al. [8] listed elastic solutions for an elbow, which are essentially the same as Goodall's result.

According to Goodall [1], only the hoop stress is affected by bend curvature. The hoop stress for a thin-walled elbow is expressed by that for a straight pipe, multiplied by a factor, which depends on the bend radius-to-pipe radius ratio and the circumferential location. The value of the factor is always less than unity, and increases with increase in bend radius-to-pipe radius ratio. Regarding the circumferential location, the hoop stress varies with position; with a maximum value at the intrados and a minimum at the extrados. Two points are also worth noting with respect to Goodall's elastic solution. The first is that his solution is for a thin-walled elbow, and thus needs to be extended to a thick-walled elbow. The second, and more important, point is that Goodall's solution does not depend on the longitudinal position in an elbow. As an elbow is always connected to straight pipes in practise, stresses should vary with the longitudinal location.

In this paper, systematic elastic finite element (FE) analysis is performed to resolve the above two issues, that is, to extend Goodall's solution to thick-walled elbows and to estimate the longitudinal variations of the elastic stresses. Section 2 describes the FE analyses performed in this work. Elastic stresses for an elbow without an attached straight pipe are given in Section 3. Section 4 reports circumferential and longitudinal elastic stresses for an elbow with an attached straight pipe. Concluding remarks are made in Section 5.

2. Finite element analysis

The main objective of this paper is to determine elastic stress distributions for an elbow under internal pressure. Although some approximations are used, determination of stress distributions is mainly based on 3-D elastic FE results.

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doi:10.1016/j.ijmecsci.2011.04.009