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# A reduced order model based on sector mistuning for the dynamic analysis of mistuned bladed disks

## P. Vargiu\*, C.M. Firrone, S. Zucca, M.M. Gola

Department of Mechanics, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129, Torino, Italy

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### ABSTRACT

In this paper, a pre-existing reduction technique suitable for the analysis of mistuned bladed disk dynamics, the Component Mode Mistuning technique (CMM), originally developed exclusively for the use of blade frequency mistuning pattern, is extended in order to allow for the introduction of a sector frequency mistuning pattern. If either mistuning is not confined to the blades (i.e. blades-to-disk interface mistuning), or the blades can not be removed from the bladed disk (i.e. integral bladed disks), sector mistuning rather than blade mistuning is a more suitable choice to perturb the tuned system. As a consequence, the extension of the original technique is referred as Integral Mode Mistuning (IMM). After a theory review of the original technique, the modifications leading to the IMM are described. Finally, the proposed IMM technique is validated in terms of both modal parameters estimation and forced response calculation, by means of a dummy bladed disk developed at Politecnico di Torino.

#### 1. Introduction

An industrial rotor generally is a very complex structure. Using a complete finite element model (FEM), in order to predict the dynamic behaviour and the stress outline of the bladed disk. is often a formidable task, due to the high computational cost needed by the calculation (an industrial FE model easily consists of millions of nodes). Consequently, a model simplification is usually requested, without any loss of accuracy; this target is reached noting that a bladed disk can be subdivided in sectors, everyone ideally equal to each other. This property, called cyclic symmetry, allows to analyse the whole structure by using only the FE model of the fundamental sector, following for instance the approach proposed by Mead [2] and Thomas [3]. The dynamic behaviour of such structures is mainly characterized by the presence of double natural frequencies, corresponding to twin mode shapes counterrotating along the system. Although a bladed disk is generally designed in order to reproduce the perfect cyclic symmetry conditions, the real rotors are always characterized by random deviations among the sectors, caused by manufacturing tolerances, material defectiveness, non-uniform assembly or wear. This issue is commonly called *mistuning*, which destroys the cyclic symmetry of the structure. The effects of the introduction of irregularities on a tuned structure have been extensively studied by the research community from the early 60s [1,4,5,6,8,9,10]: even though

mistuning is typically small, it causes the split of the double natural frequencies, a distortion of the mode shapes and, above all, mistuned bladed disks can experience drastically larger forced response levels than the ideal, tuned design, as outlined by Whitehead in [13] and MacBain et al. in [7]. The unexpected increase in maximum stresses can lead to premature high cycle fatigue (HCF) of the blades. It is clearly of great interest to be able to predict the maximum blade response as a result of mistuning. The most used approach to analyse mistuning effects on bladed disks is a statistical method, through Monte Carlo simulations, in order to evaluate the maximum possible blade forced response, as highlighted in [20]. It is well known that the accuracy obtainable from Monte Carlo techniques improves as the number of iterations grows, especially in case of rare events. This commonly makes unusable the FEM of the whole structure, due to the tremendously high computational cost required for the analysis. From this considerations the development of purpose-made FEM reduction techniques is necessary, in order to obtain more compact, but also accurate, models.

Introducing the mistuning definition, we usually refer to the socalled *geometrical mistuning*, i.e. the physical perturbations occurring in the real structural parameters. Clearly, the introduction of a geometrical mistuning into the reduced-order models (ROMs) means that all the degrees of freedom (dofs) where mistuning is present must be retained. Moreover, especially when one wants to analyse a real, manufactured bladed disk through a ROM, FEM or lumped mass model, there is usually no way to measure or determine the geometrical mistuning in the structure. An alternative mistuning model has to be considered: the *frequency mistuning*. In fact

<sup>\*</sup> Corresponding author. Tel.: +390110906953; fax: +390110906999 *E-mail address:* paolo.vargiu@email.it (P. Vargiu).

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