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## Interlaminar stress analysis of general composite laminates

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## 1. Introduction

Vast use of composite laminates in the last decades has involved engineers with new problems. One of the important problems in this issue is delamination that can occur in the vicinity of the free edges, which may commence interlaminar failures. These failure modes often happen at loading levels much lower than the prophecy of the classical lamination theory (CLT). In fact, the state of stresses in regions near the free edges, known as the boundary-layer regions, is logically a localized threedimensional field, which is naturally not estimated by CLT. However, because of inherent complexities involved in the problem no exact solution is known for elasticity equations. So, a variety of analytical and numerical methods for finding out the interlaminar stresses are developed to describe the interlaminar stresses at the free edges of composite laminates. Complete literature surveys on this subject are available in review articles of Kant and Swaminathan [1] which clearly shows the detailed path of development of methods.

Several approximately analytical studies have been accessible on the interlaminar stresses in laminated composite plates. The first approximate analysis of interlaminar stresses appear to be that of Puppo and Evensen [2] who studied interlaminar shear stresses in an idealized laminate consisting of orthotropic layers separated by isotropic shear layers with interlaminar normal stress being neglected through the laminate. This initial study has been followed by many researchers for some years. Other approximate analytical methods used to study the problem are

## ABSTRACT

In this study, based on the reduced from of elasticity displacement field for a long laminate, an analytical method is established to exactly obtain the interlaminar stresses near the free edges of generally laminated composite plates under the extension and bending. The constant parameters, which describe the global deformation of a laminate, are properly computed by means of the improved first-order shear deformation theory. Reddy's layerwise theory is subsequently utilized for analytical and numerical examinations of the boundary layer stresses within arbitrary laminated composite plates. A variety of numerical results are obtained for the interlaminar normal and shear stresses along the interfaces and through the thickness of laminates near the free edges. Finally the effects of end conditions of laminates on the boundary-layer stress are examined.

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the usage of the higher-order plate theory by Pagano [3], the perturbation technique by Hsu and Herakovich [4], the boundary layer theory by Tang and Levy [5], and the approximate elasticity solutions by Pipes and Pagano [6]. An approximate theory is also utilized by Pagano [7] based on assumed inplane stresses and the use of Reissner's variational principle. Wang and Choi [8] employed Lekhnistskii's stress potential and the theory of anisotropic elasticity for examining the free edge singularities. A variational approach concerning Lekhnitskii's stress functions is used by Yin [9] for the evaluation of free-edge stresses in laminates under uniaxial extension, bending, and torsion. An elasticity formulation is developed by Tahani and Nosier [10] for general cross-ply laminates under extension and layerwise temperature distribution. They employed the layerwise theory (LWT) to investigate the interlaminar stresses close to the free edges of the laminates. Nosier and Bahrami [11] used the reduced elasticity displacement field of a long laminated composite plate to study interlaminar stresses in antisymmetric angle-ply laminates under extension and torsion. The layerwise theory (LWT) and improved first-order shear deformation theory (IFSDT) is employed by Nosier and Maleki [12] to analyze Free-edge stresses in general composite laminates under extension loads, recently. Robbins and Reddy [13] utilized a displacement-based variable kinematic global-local finite element method. Other important methods used to study the problem are a three-dimensional finite difference solution by Altus et al. [14], a three-dimensional multilayer higher-order finite element by Gaudenzi et al. [15], and a two-dimension-three-dimension global-local finite element approach by Thomson and Griffin [16].

From the literature review it appears that very limited publications have been dedicated to study the interlaminar stresses subjected to mixed loads. In the present paper, by use of the

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