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## Calculation of forming limit diagrams using Hill's 1993 yield criterion

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## ABSTRACT

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Keywords: Forming limit diagram Yield criterion Strain hardening Strain rate sensitivity Based on the analysis proposed by Jones and Gillis (JG), forming limit diagrams (FLDs) are calculated from idealization of sheet deformation into three stages: (I) homogeneous deformation up to maximum load, (II) deformation localization under constant load, and (III) local necking with precipitous drop in load. In the calculation, Hill's 1993 yield criterion is used. Using this yield criterion and the JG model, effects of materials parameters such as ratio of uniaxial to equi-biaxial yield stress, strain hardening, strain rate sensitivity and plastic anisotropy on the shape and level of forming limit curves are studied. In addition, the capability of the JG model to predict limit strains is demonstrated through comparison of calculated results with experimental data for the interstitial free (IF) steel and aluminum alloys 3003-0 and 8014-0. It is concluded that although the model predicts the effect of material parameters reasonably well, the calculated limit strains are higher than the experimental FLDs. The observed discrepancy may be attributed to the overestimation of planar isotropy, cavitation and the nature of texture present in the sheets. Due to the overestimation of the predictions, care must be taken when using this approach for industrial purposes.

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## 1. Introduction

Forming Limit Diagrams (FLDs), first proposed by Keeler [1] and Goodwin [2] are generally used as a diagnostic tool for sheet metal formability over various loading paths. They show the limit strains the material can sustain in different modes of deformation, including drawing and stretching states. While the experimental construction of the FLD is well established in the literatures [1–4], many attempts have been made to numerically calculate the FLDs, which might be less time consuming in die design and trouble-shooting processes. The main concept in the modeling sheet metal response in different processes is how the material yields and fails in a specific condition. Based on sheet metal properties and process parameters, different approaches have been used for the explanation of necking phenomenon and the calculation of FLDs. The method used by Marciniak and Kuczynski [5] may be the most prevalent model which is considered in this field of study. In their model, the assumption of the existence of an initial imperfection as a groove in the sheet plane and its behavior compared with the adjacent regions in the material is considered for the simulation of what happens in reality. Many researchers have tried to apply this model in conjunction with different yield criteria and hardening laws. The results show the dependency of the calculation on the used yield criteria, constitutive equations, as well as the initial size of the assumed imperfection in the material [6–10]. There have been some attempts to consider the effect of cavitation and void growth in prediction of FLDs. Kim and Kim [11] incorporated the void growth concept in the M-K method to predict the forming limit diagram of steel sheets. They used the Avrami-like equation for relating density changes due to void growth and strain during stage II but they could not find reasonable results and found that the best coincidence between analysis and experiment was obtained by assuming an unrealistic value of the inhomogeneity factor or Avrami-like parameters. Date and Padmanabhan [12] used the same approach for considering the effect of cavitation on prediction of the FLD of steel, Al-Mg, Al-Cu-Mg and Al-Mn sheets. They found a good correlation between the experimentally determined and predicted FLDs of steel by combining M-K theory with Hutchinson-Tvergaard equation for all strain ratios, or using the same equation for negative strain ratios. They concluded that satisfactory predictions could be obtained once the deformation and fracture behavior of a material are known. There are clear evidences showing that the assumption of an imperfection for occurrence of failure in sheet metals is not necessary and therefore, analyzing limit strains that are independent of such a presumption would be valuable. This point of view was initiated by Gillis and Jones [13] who proposed their theory

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