



# The mathematical description of the body centre of mass 3D path in human and animal locomotion

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## ABSTRACT

Although the 3D trajectory of the body centre of mass during ambulation constitutes the 'locomotor signature' at different gaits and speeds for humans and other legged species, no quantitative method for its description has been proposed in the literature so far. By combining the mathematical discoveries of Jean Baptiste Joseph Fourier (1768–1830, analysis of periodic events) and of Jules Antoine Lissajous (1822–1880, parametric equation for closed loops) we designed a method simultaneously capturing the spatial and dynamical features of that 3D trajectory. The motion analysis of walking and running humans, and the re-processing of previously published data on trotting and galloping horses, as moving on a treadmill, allowed to obtain closed loops for the body centre of mass showing general and individual locomotor characteristics. The mechanical dynamics due to the different energy exchange, the asymmetry along each 3D axis, and the sagittal and lateral energy recovery, among other parameters, were evaluated for each gait according to the present methodology. The proposed mathematical description of the 3D trajectory of the body centre of mass could be used to better understand the physiology and biomechanics of normal locomotion, from monopods to octopods, and to evaluate individual deviations with respect to average values as resulting from gait pathologies and the restoration of a normal pattern after pharmacological, physiotherapeutic and surgical treatments.

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## 1. Introduction

All people walk in a similar way (this applies also to running or skipping) but we are able to promptly detect slight deviations from the common movement pattern and sometimes recognise through this someone we know. In particular, what we mainly spot are changes in the motion of the head/trunk segment, which reflect changes in the 3D path of the body centre of mass (BCOM). Apart from anecdotal and subjective evaluation of gait, there are a few professional categories specifically interested in the quantitative assessment of locomotion:

(1) Biomechanists have classically been interested in general features of gaits, such as the mechanical work necessary to sustain locomotion and the energy saving strategies to contain the metabolic cost of transport (e.g. Cavagna and Margaria, 1966; Alexander, 1989). Both aspects have been found to mostly relate to the 3D trajectory of BCOM, whose time-course (periodic raises and accelerations) affects the so-called 'external' mechanical work.

- (2) Locomotion pathologists have commonly used inverse dynamics to estimate joint moments and powers, and compare them to standard values. It is intuitive, though, that the different types of *claudicatio* could be investigated, at the first glance, as deviations of BCOM trajectory from the 'normal' 3D path, and the success of a rehabilitative programme should be ultimately evaluated on the basis of the restoration of a healthy, almost symmetrical BCOM path.
- (3) Comparative physiologists and theoretical biologists can use BCOM path description to infer the influence of basic bipedal locomotor paradigms (walking, hopping, and skipping) in animal evolution when the number of legs varies (from monopods to octopods).
- (4) Biomedical and automation engineers get insights about the efficacy of locomotion, for bipedal/quadrupedal robots, from a quantitative analysis of the different gaits. The results obtained through a biomimetic design of legged machines can be tested by comparing their motion to the one of true biological 'vehicles'.

Thus there are many reasons to develop a clear methodological framework devoted to mathematically describe the experimentally captured motion of BCOM in 3D. A purposely designed set of equations would summarise both the general aspects of the gaits

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