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## Tutorial Review

# Hilbert transform in vibration analysis

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## ABSTRACT

This paper is a tutorial on Hilbert transform applications to mechanical vibration. The approach is accessible to non-stationary and nonlinear vibration application in the time domain. It thrives on a large number of examples devoted to illustrating key concepts on actual mechanical signals and demonstrating how the Hilbert transform can be taken advantage of in machine diagnostics, identification of mechanical systems and decomposition of signal components.

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