Contents lists available at ScienceDirect



Mechanical Systems and Signal Processing



journal homepage: www.elsevier.com/locate/jnlabr/ymssp

## Probabilistic uncertainty analysis of an FRF of a structure using a Gaussian process emulator

Thomas E. Fricker<sup>a,\*</sup>, Jeremy E. Oakley<sup>b</sup>, Neil D. Sims<sup>c</sup>, Keith Worden<sup>c</sup>

<sup>a</sup> Mathematics Research Institute, University of Exeter, Harrison Building, North Park Road, Exeter EX4 4QF, UK

<sup>b</sup> Department of Probability and Statistics, University of Sheffield, Sheffield S3 7RH, UK

<sup>c</sup> Department of Mechanical Engineering, University of Sheffield, Sheffield S1 3JD, UK

## ARTICLE INFO

Article history: Received 22 July 2010 Received in revised form 19 April 2011 Accepted 14 June 2011 Available online 20 July 2011

Keywords: Finite element model Probabilistic uncertainty analysis Envelope frequency response function Gaussian process Metamodel Bayesian

## ABSTRACT

This paper introduces methods for probabilistic uncertainty analysis of a frequency response function (FRF) of a structure obtained via a finite element (FE) model. The methods are applicable to computationally expensive FE models, making use of a Bayesian metamodel known as an *emulator*. The emulator produces fast predictions of the FE model output, but also accounts for the additional uncertainty induced by only having a limited number of model evaluations. Two approaches to the probabilistic uncertainty analysis of FRFs are developed. The first considers the uncertainty in the response at discrete frequencies, giving pointwise uncertainty intervals. The second considers the uncertainty in an entire FRF across a frequency range, giving an uncertainty envelope function. The methods are demonstrated and compared to alternative approaches in a practical case study.

© 2011 Elsevier Ltd. All rights reserved.

## 1. Introduction

Finite element (FE) modelling is perhaps the most widely used computational tool in the analysis of structural vibrations, particularly for the prediction of frequency response functions (FRFs). In recent years there has been a growing level of interest in how different types of uncertainty can be handled with this modelling approach. These uncertainties can be inherent to the model itself (for example due to assumptions regarding the boundary conditions), or alternatively they could arise due to unknown values of physical parameters (for example component geometry or material properties). In the latter case, this lack of knowledge could be attributed to variation between nominally identical components (i.e. variability), or uncertainty during the design process regarding the final choice of dimensions or material.

There has long been interest in how uncertainty propagates through FE models. The method with the greatest pedigree is the stochastic finite element (SFE) method [1]; this is a probabilistic method. In the general SFE formulation, the material properties across the structure can be specified as a random field. In a manner similar to the discretisation of the structure into finite elements, the random field is discretised into a denumerable set of random variables using the Karhunen–Loeve expansion, which is then truncated at some finite order. The results from the FE model are then expressed as a mean value supplemented by an expansion in terms of the random variables, allowing statistics of the quantity of interest to be computed. In the last decade, interest has grown in possibilistic approaches, such as a fuzzy approach to FE analysis and computation of modal quantities [2,3]. More recent work has considered component mode synthesis as a framework for

<sup>\*</sup> Corresponding author. Tel.: +44 1392 723628; fax: +44 1392 217965.

E-mail addresses: frickertom@hotmail.com, t.fricker@ex.ac.uk (T.E. Fricker), j.oakley@shef.ac.uk (J.E. Oakley), n.sims@sheffield.ac.uk (N.D. Sims), k.worden@sheffield.ac.uk (K. Worden).

<sup>0888-3270/\$ -</sup> see front matter  $\circledcirc$  2011 Elsevier Ltd. All rights reserved. doi:10.1016/j.ymssp.2011.06.013