Mechanical Systems and Signal Processing 25 (2011) 2358–2374

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/jnlabr/ymssp

Time–frequency characterization of nonlinear normal modes and challenges in nonlinearity identification of dynamical systems

P. Frank Pai *

Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA

A R T I C L E I N F O

Article history:
Received 19 July 2010
Received in revised form
25 December 2010
Accepted 22 February 2011
Available online 3 March 2011

Keywords:
Nonlinear normal modes
Instantaneous frequency and amplitude
Time–frequency analysis
Conjugate-pair decomposition
Hilbert–Huang transform

A B S T R A C T

Presented here is a new time–frequency signal processing methodology based on Hilbert–Huang transform (HHT) and a new conjugate-pair decomposition (CPD) method for characterization of nonlinear normal modes and parametric identification of nonlinear multiple-degree-of-freedom dynamical systems. Different from short-time Fourier transform and wavelet transform, HHT uses the apparent time scales revealed by the signal's local maxima and minima to sequentially sift components of different time scales. Because HHT does not use pre-determined basis functions and function orthogonality for component extraction, it provides more accurate time-varying amplitudes and frequencies of extracted components for accurate estimation of system characteristics and nonlinearities. CPD uses adaptive local harmonics and function orthogonality to extract and track time-localized nonlinearity-distorted harmonics without the end effect that destroys the accuracy of HHT at the two data ends. For parametric identification, the method only needs to process one steady-state response (a free undamped modal vibration or a steady-state response to a harmonic excitation) and uses amplitude-dependent dynamic characteristics derived from perturbation analysis to determine the type and order of nonlinearity and system parameters. A nonlinear two-degree-of-freedom system is used to illustrate the concepts and characterization of nonlinear normal modes, vibration localization, and nonlinear modal coupling. Numerical simulations show that the proposed method can provide accurate time–frequency characterization of nonlinear normal modes and parametric identification of nonlinear dynamical systems. Moreover, results show that nonlinear modal coupling makes it impossible to decompose a general nonlinear response of a highly nonlinear system into nonlinear normal modes even if nonlinear normal modes exist in the system.

1. Introduction

For a linear dynamic system of n degrees of freedom (DOF) governed by

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F\}, \tag{1a}$$

* Tel.: +1 573 884 1474; fax: +1 573 884 5090.
E-mail address: paip@missouri.edu

0888-3270/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ymssp.2011.02.013