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# On the properness condition for modal analysis of non-symmetric second-order systems

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### ABSTRACT

Non-symmetric second-order systems can be found in several engineering contexts, including vibroacoustics, rotordynamics, or active control. In this paper, the notion of properness for complex modes is extended to the case of non-self-adjoint problems. The properness condition is related to the ability of a set of complex modes to represent in an exact way the behavior of a physical second-order system, meaning that the modes are the solutions of a quadratic eigenvalue problem whose matrices are those of a physical system. This property can be used to identify the damping matrices which may be difficult to obtain with mathematical modeling techniques. The first part of the paper demonstrates the properness condition for non symmetric systems in general. In the second part, the authors propose a methodology to enforce that condition in order to perform an optimal reconstruction of the "closest" physical system starting from a given basis complex modes. The last part is dedicated to numerical and experimental illustrations of the proposed methodology. A simulated academic test case is first used to investigate the numerical aspects of the method. A physical application is then considered in the context of rotordynamics. Finally, an experimental test case is presented using a structure with an active control feedback. An extension of the LSCF identification technique is also introduced to identify both left and right complex mode shapes from measured frequency response functions.

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### 1. Introduction

Experimental modal analysis is a very commonly employed tool in structural dynamics. Indeed, given the identified modal parameters (eigenfrequencies, eigenvectors, modal damping ratios and modal masses), the dynamic response of a structure to an arbitrary excitation can be readily evaluated at the measured degrees of freedom. The experimentally identified eigensolutions can also be used to construct a reduced dynamic model which conserves some of the physical properties of the system, for example the system topology, mass, and stiffness characteristics. While the mass and stiffness properties can often be accurately represented with mathematical modeling techniques (e.g. finite element analysis), the mechanisms of energy dissipation remain notoriously difficult to simulated numerically and the present paper focuses on this particular point.

It has been shown [1] that for structural dynamics problems with symmetric matrices, the existence of an equivalent physical experimental reduced model is equivalent to the so-called properness condition of complex vectors. Moreover,

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