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A signal decomposition or lowpass filtering with Hilbert transform?

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ABSTRACT

Recently, Chen and Wang discovered an explicit formula that makes use of the Hilbert transform for accurate decomposition of a lower harmonic from a signal composition. This letter presents another proof with a new interpretation for the formula using the Bedrosian identity for overlapping signals. This new and simpler proof is based only on the Hilbert transform and does not involve presentation of the Fourier transform. As a result the discovered formula is introduced as a lowpass filter suitable for non-stationary signals.

1. Introduction

A recent paper [1] proposes a new and interesting idea, called the "decomposition theorem", for extraction of a single low frequency harmonic component from a time domain vibration composition. The idea is based on the product of the composition $x(t)=x_{\text{high}}+s_{\text{slow}}$ with two orthogonal harmonics having a fixed "bisecting" frequency ω_b: $\omega_{x_{\text{slow}}}<\omega_b<\omega_{x_{\text{high}}}$. Both orthogonal products are further transformed with the Hilbert transform and finally are multiplied with the replaced orthogonal "bisecting" harmonics (3 [1]): $s_{slow}(t)=\sin(\omega_{\text{b}}t)H[x(t)\cos(\omega_{\text{b}}t)]-\cos(\omega_{\text{b}}t)H[x(t)\sin(\omega_{\text{b}}t)]$. The extracted slow harmonic $s_{\text{slow}}(t)$ preserves its initial amplitude, frequency and phase relations. This formula is new, simple and very original. Yet it relates only to a single harmonic component and a constant "bisecting" frequency. The proof of the formula also is based on the Fourier transform models and is rather cumbersome.

This letter offers a new proof of the found formula using a modified Bedrosian identity (product theorem) for a product of real functions with overlapping spectra [2,3]. Let $n(t)$ and $x(t)$ be fast varying functions whose frequency bands do overlap. If one of the functions can be represented in the form of the sum of two parts $n(t)=\overline{n}_{\text{slow}}(t)+\overset{\leftrightarrow}{n}_{\text{fast}}(t)$, then the HT of the product of these functions with overlapping spectra can also be written in the form of the sum of two parts [3]:

$$H[n(t)x(t)]=H\{[\overline{n}_{\text{slow}}(t)+\overset{\leftrightarrow}{n}_{\text{fast}}(t)]x(t)\}=\overline{n}_{\text{slow}}(t)\tilde{x}(t)+\tilde{n}_{\text{fast}}(t)x(t) \tag{1}$$

where $\overline{n}_{\text{slow}}(t)$ is the slow (lowpass) part of the real function, $\overset{\leftrightarrow}{n}_{\text{fast}}(t)$ is the fast (highpass) part and $\tilde{n}_{\text{fast}}(t)$ is the HT pair component of the fast component $\overset{\leftrightarrow}{n}_{\text{fast}}(t)$.

2. Generalized slow component detection

Let us assume that the composition $x=s(t)+f(t)$ is a real function with slow $s(t)$ and fast components $f(t)$ with non-overlapping spectra. Assume also that $Y=y+i\tilde{y}$ is a complex function with an intermediate spectrum that lies between the spectra of the slow and the fast component. Here $\tilde{y}(t)$ is the Hilbert transform projection of $y(t)$. An interesting result obtained in [1] can be rewritten in a more general and compact way:

$$\text{Re}[H(xY)]\text{Re}\tilde{Y}-\text{Im}[H(xY)]\text{Im}\tilde{Y}=s(t) \tag{2}$$

3. Proof

The new proof of (2) is much shorter and simpler. The first term of the composition on the left hand side can be transformed according to (1) $\text{Re}[H(xY)]\text{Re}\tilde{Y}=(s\tilde{y}+\tilde{f}y)\tilde{y}=s\tilde{y}^2+\tilde{f}y\tilde{y}$. The second term takes the following simpler form: $\text{Im}[H(xY)]\text{Im}\tilde{Y}=H[(s+f)y]\tilde{y}=H[(s+f)\tilde{y}]y=(-sy+\tilde{f}\tilde{y})y=-sy^2+\tilde{f}y\tilde{y}$. Therefore, their algebraic sum is$s\tilde{y}^2+\tilde{f}y\tilde{y}-(-sy^2+\tilde{f}y\tilde{y})=s(t)$, which is what we set out to prove.

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