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Numerical simulation of propagation of solitary deformation waves in a compressible hyperelastic rod

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Abstract

In this paper, propagation of axisymmetric deformation waves in circular cylindrical rods of compressible hyperelastic material is studied. The corresponding model equation that describes propagation of waves of moderate wave-length takes into account the coupling effect of the material nonlinearity and the geometric size of the rod. The model equation is integrated numerically under sech^2-type initial conditions making use of the discrete Fourier transform based pseudospectral method. Numerical simulations are carried out over a wide range of material parameters. The analysis of the time–space behaviour of solutions demonstrates that in some domains of space of material parameters single solitary wave solutions or trains of interacting solitons can be emerged from the initial localised pulses.

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1. Introduction

Nonlinear axial–radial deformation waves in hyperelastic rods have been studied by many authors (see, for example, Samsonov [23], Porubov and Samsonov [20], Dai and Huo [6], and Dai and Fan [8]). These studies concentrated on weakly nonlinear long waves and only smooth solutions were considered. However, there are situations when the wave-length is only of moderate length. For these cases, it was argued in Dai [7] and Dai and Huo [9] that it is necessary to take into account the coupling effect of the material nonlinearity and the geometric size of the rod. With such a consideration, a new type of nonlinear equation with nonlinear dispersion terms was derived in Dai [7] and Dai and Huo [9] for a compressible Mooney–Rivlin material and a general compressible hyperelastic material respectively (compressible rubber and rubber-like materials, for example). The equation takes the following form:

$$u_\tau + \sigma_1 u u_\xi + \sigma_2 u_{\xi\xi\tau} + \sigma_3(2u_\xi u_{\xi\xi} + u u_{\xi\xi\xi}) = 0. \tag{1}$$

Here the dimensionless independent variables ξ and τ correspond to the moving frame, u is a quantity measuring the radial deformation and σ_1, σ_2 and σ_3 are related to the material constants and the initial stretch of the rod. The last

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