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Original article

Numerical solution of the 'classical' Boussinesq system

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Abstract

We consider the 'classical' Boussinesq system of water wave theory, which belongs to the class of Boussinesq systems modelling two-way propagation of long waves of small amplitude on the surface of water in a horizontal channel. (We also consider its completely symmetric analog.) We discretize the initial-boundary-value problem for these systems, corresponding to homogeneous Dirichlet boundary conditions on the velocity variable at the endpoints of a finite interval, using fully discrete Galerkin-finite element methods of high accuracy. We use the numerical schemes as exploratory tools to study the propagation and interactions of solitary-wave solutions of these systems, as well as other properties of their solutions.

Keywords: Water waves; 'Classical' Boussinesq systems; Initial-boundary-value problems; Fully discrete Galerkin-finite element methods; Solitary waves

1. Introduction

We consider the so-called 'classical' Boussinesq system (CB)

$$\begin{aligned} &\eta_t + u_x + (\eta u)_x = 0, \\ &u_t + \eta_x + u u_x - \frac{1}{3} u_{xxt} = 0, \end{aligned} \tag{1.1}$$

for $x \in \mathbb{R}$, $t > 0$, supplemented by the initial conditions

$$\eta(x, 0) = \eta_0(x), \qquad u(x, 0) = u_0(x), \tag{1.2}$$

where η_0, u_0 are given real functions on $\mathbb{R}$. The system (1.1) is a Boussinesq-type approximation of the two-dimensional Euler equations that models two-way propagation of long waves of small amplitude on the surface of an incompressible, inviscid fluid in a uniform horizontal channel of finite depth. The variables in (1.1) and (1.2) are nondimensional and unscaled: x and t are proportional to position along the channel and time, respectively, and $\eta(x, t)$ and $u(x, t)$ are proportional to the elevation of the free surface above the level of rest $y = 0$, and to the horizontal velocity of the fluid at a height $y = -1 + (1 + \eta(x, t))/\sqrt{3}$, respectively. (In terms of these variables the bottom of the channel is at $y = -1$.)

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