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On the numerical discretisation of stochastic oscillators

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Abstract

In this article, we propose an approach, based on the variation-of-constants formula, for the numerical discretisation over long-time intervals of several stochastic oscillators. Additive and multiplicative noises are treated separately. The proposed schemes permit the use of large step sizes in the presence of a high frequency in the problem and offer various additional properties. These new numerical integrators can be viewed as a stochastic-generalisation of the trigonometric integrators for highly oscillatory deterministic problems.

Keywords: Stochastic oscillators; Highly oscillatory problems; Kubo oscillator; Langevin equation; Numerical schemes; Stochastic trigonometric integrators

1. Introduction

In this article, we propose an approach based on the variation-of-constants formula for the numerical discretisations of various stochastic oscillators and show that these new schemes possess properties shared by the exact solution of our second order SDE model problems.

We will focus our numerical analysis on a scalar model of a stochastic oscillator with additive noise and with a high frequency. In this case, we will prove uniform error bounds (i.e. independent of the large parameter in our problem) and also show that the proposed schemes offer various additional properties. But it turns out that very similar ideas used for the above case can be applied to derive numerical schemes for other problems that are different in nature too. We will thus briefly discuss two possible extensions to other second order SDEs: the Kubo oscillator and the Langevin equation. For these two equations, we will present some numerical experiments in order to demonstrate that these new integrators are robust and qualitatively correct.

Let us first of all describe in details the stochastic oscillators we will consider.

In Section 2, we will deal with oscillators with additive noise and with a high frequency. As a prototype equation, we will consider the nonlinear stochastic oscillator

$$\ddot{X}_t + \omega^2 X_t = g(X_t) + \alpha \dot{W}_t, \tag{1}$$

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