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Continuation of oscillatory patterns in a road traffic model of three cars

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Abstract

We investigate microscopic models of the road traffic. In particular, we consider the car-following model for a single-line traffic flow of N identical cars on a circular road. The classical differentiable model breaks down at the time instant when two cars collide. Nevertheless, the natural action of a driver would be to overtake the slower car. In our previous work, we proposed the model which simulates overtaking. We observed a large variety of oscillatory solutions (*oscillatory patterns*) of the model. In the present contribution, we assume $N=3$ i.e., three cars on the route, and formulate our model as a particular piecewise smooth (PWS) dynamical system. We define generic oscillatory patterns as invariant objects of the flow defined by this PWS system. We use the standard software to continue the patterns with respect to a parameter (the length of the route). In this respect, we also investigate bifurcation phenomena.

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1. Introduction

The aim of this contribution is to present some new results of the analysis of a road traffic model with overtaking. For a general reference to the modeling of traffic flows, see [15]. We consider a particular *microscopic* model of the traffic flow on a circular road of length L in the following form

$$x_i' = y_i, \tag{1a}$$

$$y_i' = \frac{1}{\tau}\left[V(x_{i+1}-x_i)-y_i\right], \quad x_{N+1}=x_1+L, \tag{1b}$$

$i=1,\ldots,N$, where N is the number of cars, see [3]. Each pair $(x_i, y_i)\in\mathbb{R}^2$ defines the position and the velocity of the i th car. The model depends on the choice of a nondecreasing function V which is called the *optimal velocity function*.

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