

Original Articles

Numerical stability of symmetric solitary-wave-like waves of a two-layer fluid—Forced modified KdV equation

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Abstract

Forced internal waves at the interface of a two-layer incompressible fluid in a two-dimensional domain with rigid horizontal boundaries are studied. The lower boundary is assumed to have a small obstruction. We derive a time-dependent forced modified KdV equation when the KdV theory fails and study the stabilities of four types of symmetric time-independent solitary-wave-like solutions numerically.

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1. Introduction

There have been great interests in studying interfacial waves of a fluid and many interesting features of the waves have been found by different researchers. Steady waves of two-layer fluids have been studied numerically by Sha and Vanden-Broeck [16], Forbes [10], Dias and Vanden-Broeck [8], among others, and asymptotically by Choi, Sun, and Shen [6,7], Choi [5], Shen, Monohar and Gong [18], and many more. A rigorous justification of the formal asymptotic method in a perfect gas was given by Shen and Sun [17]. Sha and Vanden-Broeck [16] showed no solitary wave appears in an immiscible two layer incompressible fluid if gravity is not concerned. Gong, Monohar and Shen derived a forced KdV equation asymptotically. They found two types of steady symmetric solitary-wave-like solutions numerically from a forced KdV equation and studied the time stability of them. Choi, Sun, and Shen derived a steady forced modified KdV equation when KdV theory fails. By studying the forced modified KdV equation, they discovered many interesting wave patterns among which three types of steady symmetric solitary-wave-like solutions were numerically found. Choi [5] found one more type of steady symmetric solitary-wave-like solution numerically. Larkin [14] proved the existence and uniqueness of strong and weak global solutions for the modified KdV equation with a source term in a bounded domain as well as the exponential decay of solutions.

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