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Original article

Characteristic mixed finite element approximation of transient convection diffusion optimal control problems ☆

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Abstract

In this paper, we investigate a characteristic mixed finite element approximation of transient convection diffusion optimal control problems. The state and the adjoint state are approximated by characteristic mixed finite element method, while the control is discretized by standard finite element method. We derive the continuous and discrete first-order optimality conditions and prove a priori error estimates for the state, the adjoint state and the control. Numerical examples are presented to illustrate the theoretical findings.

Keywords: Characteristic mixed finite element; Optimal control; Transient convection diffusion equations; A priori error estimates

1. Introduction

Optimal control problems governed by convection diffusion equations play an important role in many scientific and engineering problems [14,23]. There exist many contributions to numerical methods and algorithms for this kind of problems. We refer to [2,20,21] for stabilized finite element methods and [11,22] for discontinuous Galerkin finite element methods. For more works the readers may refer to the references cited therein.

As we know that transient convection diffusion problem often admits solution with steep moving front and complicated structures. Classic numerical methods tend to generate numerical solutions with spurious oscillations, excessive numerical diffusion, or a combination of both. Extensive research has been carried out on the development of efficient and effective numerical methods for the solution of transient convection diffusion equations. One important class of methods is the characteristic method [7], which takes the advantage of the hyperbolic nature of the governing equations in the numerical discretizations. Some characteristic methods were developed based on the primal form of the governing equations, such as the characteristic Galerkin method [13,17], the modified method of characteristics [7,19], the Eulerian–Lagrangian localized adjoint method [3,18], and so on. While other characteristic methods were developed

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