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Original article

# Lanczos–Chebyshev pseudospectral methods for wave-propagation problems

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**Abstract**

The pseudospectral approach is a well-established method for studies of the wave propagation in various settings. In this paper, we report that the implementation of the pseudospectral approach can be simplified if power-series expansions are used. There is also an added advantage of an improved computational efficiency. We demonstrate how this approach can be implemented for two-dimensional (2D) models that may include material inhomogeneities. Physically relevant examples, taken from optics, are presented to show that, using collocations at Chebyshev points, the power-series approximation may give very accurate 2D soliton solutions of the nonlinear Schrödinger (NLS) equation. To find highly accurate numerical periodic solutions in models including periodic modulations of material parameters, a real-time evolution method (RTEM) is used. A variant of RTEM is applied to a system involving the copropagation of two pulses with different carrier frequencies, that cannot be easily solved by other existing methods.


*Keywords:* Pseudospectral Chebyshev method; Nonlinear Schrödinger equations; Waves in complex media; Solitary wave propagation; Real-time evolution method

## 1. Introduction

Recent developments in photonics and studies of nanostructured materials stimulate the interest in developing efficient and accurate algorithms for numerical solutions of models similar to the nonlinear Schrödinger (NLS) equation. In many cases, the spectral approach [6,24,25], implemented in the conjunction with the collocation procedure in the framework of pseudospectral methods [18,22], has been involved. Typical examples are the Fourier pseudospectral method [22,25], the Chebyshev pseudospectral (CPS) method [5,25], the Legendre pseudospectral method [16,25] and others, which use different orthogonal polynomials, such as Lagrange polynomials [33]. The pseudospectral approach using orthogonal polynomials is flexible in its application to complex problems, such as equations with coordinate-dependent coefficients and/or nonlinear terms [25]. Note that many wave-propagation problems may require the use of more elaborate boundary conditions [2,3]. Unlike the case of the spectral method, general boundary conditions could easily be implemented if a pseudospectral approach is utilized [20,21].

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