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Motion design with Euler–Rodrigues frames of quintic Pythagorean-hodograph curves

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Abstract

The paper presents an interpolation scheme for G^1 Hermite motion data, i.e., interpolation of data points and rotations at the points, with spatial quintic Pythagorean-hodograph curves so that the Euler–Rodrigues frame of the curve coincides with the rotations at the points. The interpolant is expressed in a closed form with three free parameters, which are computed based on minimizing the rotations of the normal plane vectors around the tangent and on controlling the length of the curve. The proposed choice of parameters is supported with the asymptotic analysis. The approximation error is of order four and the Euler–Rodrigues frame differs from the ideal rotation minimizing frame with the order three. The scheme is used for rigid body motions and swept surface construction.

Keywords: Pythagorean-hodograph; Euler–Rodrigues frame; Rotation minimizing frame; Motion design; Quaternion; Hermite interpolation

1. Introduction

To compute an orthonormal frame of a spatial curve $\boldsymbol{r}$ is an important task in computer animation, motion planning, swept surface construction, etc. Frames determine an orientation of a rigid body as it traverses the curve. Typically an adapted frame $(\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3)$ is searched for, which has the property that $\boldsymbol{f}_1 = \dot{\boldsymbol{r}}/\|\dot{\boldsymbol{r}}\| = \boldsymbol{t}$ is a tangent vector, and the remaining two vectors span the normal plane. A well known adapted frame is the Frenet frame $(\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b})$ (see [12]), but it is often unsuitable for practical applications since it is not defined at inflection points and it incurs an unnecessary rotation of the normal plane vectors $\boldsymbol{n}$ and $\boldsymbol{b}$ around $\boldsymbol{t}$. The most attractive frame in motion design applications and swept surface construction is a rotation minimizing frame (RMF frame), which is characterized through a solution of first-order differential equations (see [11], e.g.). More precisely, there should be no instantaneous rotation of $\boldsymbol{f}_2$ and $\boldsymbol{f}_3$ around $\boldsymbol{f}_1 = \boldsymbol{t}$. The variation of any adapted frame $(\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3)$ along the curve $\boldsymbol{r}$ is determined by the angular velocity vector $\boldsymbol{\omega}$ as

$$\dot{\boldsymbol{f}}_1 = \boldsymbol{\omega} \times \boldsymbol{f}_1, \quad \dot{\boldsymbol{f}}_2 = \boldsymbol{\omega} \times \boldsymbol{f}_2, \quad \dot{\boldsymbol{f}}_3 = \boldsymbol{\omega} \times \boldsymbol{f}_3.$$

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